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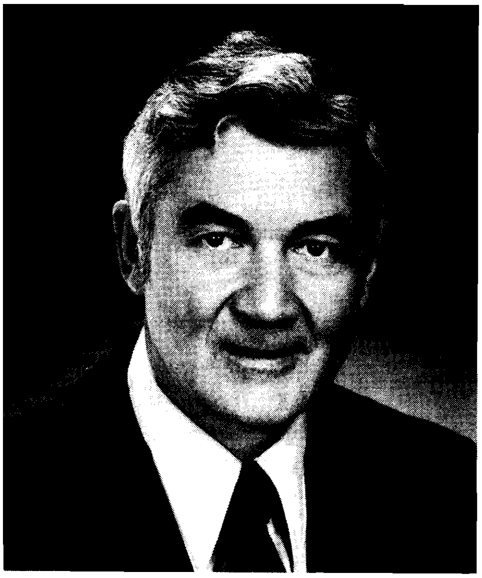
Mathematical Sciences in Canada

by A.J. Coleman, G.D. Edwards,
K.P. Beltzner

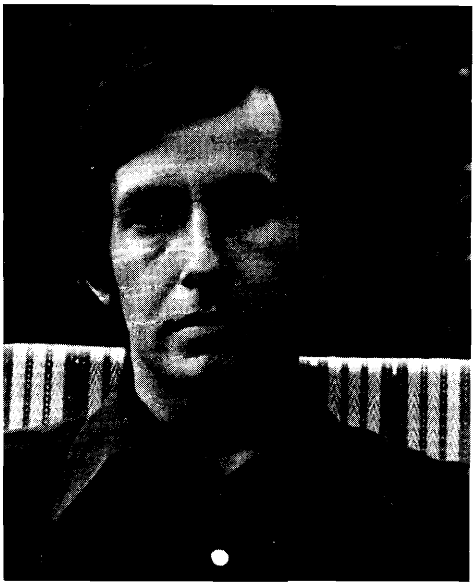
1975

Science Council of Canada
150 Kent Street
7th Floor
Ottawa, Ontario
K1P 5P4



A.J. Coleman

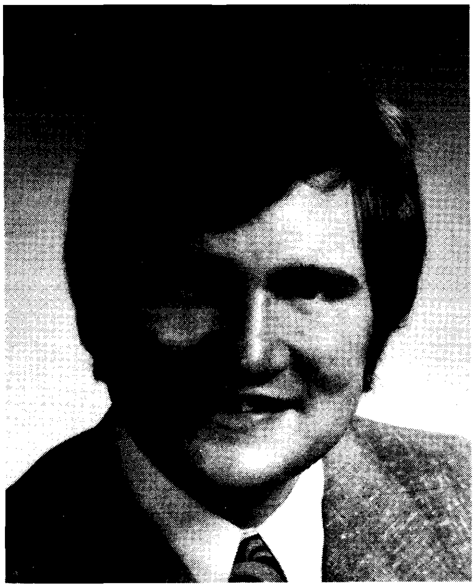
John Coleman has been the Head of the Department of Mathematics at Queen's University since 1960 and President of the Canadian Mathematical Congress from 1973 to 1975. His thesis, under Leopole Infeld was on Quantum Mechanics. His papers deal with group theory and with the N-body problem in quantum physics. Coleman chaired the Ontario Mathematics Commission for four years at the epoch when the "new math" arrived in Ontario (he claims it started in 1852 with Cayley's paper on Groups). He was one of the Editors of a series of textbooks for schools published by W.J. Gage and Company, which with subsequent series, has sold over one million copies - mostly in Ontario, but also in Hong Kong and New Zealand. In the period 1945-49, Professor Coleman served the World's Student Christian Federation, visiting over 100 universities in twenty countries throughout the world. His wife, Marie-Jeanne de Hallar, who also worked for U.S.C.F., graduated in Theology from the College founded by Calvin. She is a daughter of the Swiss banker referred to in Chapter II. Coleman's enthusiasms are always passionate. They include music, squash, sailing his Flying Dutchman, wine and mathematics. He believes that the basic problems of mankind are religions. His intellectual analysis of the future is always foreboding. His attitude to the future is always optimistic.



G. Edwards

Gordon Edwards, Assistant Director, Mathematics Study: B.Sc. (Mathematics and Physics) University of Toronto, 1961; M.S. (Mathematics) University of Chicago, 1962; M.A. (English Literature) University of Chicago, 1964; Ph.D. (Abstract Algebra) Queen's University, 1972.

Dr. Edwards has had more than ten years teaching experience at the post-secondary level. In addition to holding a faculty appointment at the University of Western Ontario (1964-68), he has also taught at the following universities: Chicago, Queen's, British Columbia, Ottawa and Sir George Williams. His diverse background includes employment as an inspector for the Attorney General of Ontario, as an actuarial assistant for Excelsior Life, as a Science Adviser for the Science Council of Canada, and as the editor of SURVIVAL Magazine - an international ecology newsletter. He has published work on Lie Algebras, Infinitesimal Group Schemes, and the Economics of Ocean Fisheries. He is presently preparing an anthology of environmentally-oriented teaching guides for high school chemistry teachers, which will be published by Concordia University. He is a faculty member of Vanier College in Montreal, and is a prominent figure in the recently-formed Canadian Coalition for Nuclear Responsibility. Among the awards received by Dr. Edwards are a Woodrow Wilson Fellowship, a Queen Elizabeth I Fellowship, and a College Gold Medal in Mathematics and Physics (Toronto). He describes himself as a teacher.



K.P. Beltzner

Klaus Beltzner was born in Bolzano, Italy, in 1947 and arrived in Montreal in 1956. He was awarded a B.Sc. degree from McGill University in 1970, and then moved to Ontario to pursue postgraduate studies at the University of Waterloo. Mr. Beltzner obtained his M.Math degree in 1971, nominally in Statistics but with strong secondary specialization in Computer Applications.

Mr. Beltzner arrived in Ottawa in 1972 and began working in the Survey Methodology Group of Statistics Canada, where he was instrumental in the introduction of APL computing skills. He was involved in the New Retail Trade Survey when he left Statistics Canada to join the Science Council in August, 1973. He is associated with the Mathematics Study in the capacity of Project Officer, and has presented papers related to the Study at conferences in Vancouver and Atlanta. Mr. Beltzner has also offered mathematical expertise to numerous other Science Council projects.

Mr. Beltzner's research interests center on the practical application of statistical knowledge and computer technology to problems besetting Canadians in our modern age. He is constantly seeking to improve survey techniques and Science Council studies have and will continue to benefit from his development of that area.

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APPENDICES

Preface to the Preliminary Edition

This Preliminary Edition has been produced in offset in order to make the report available to the officers of the sponsoring organizations and to fulfill our promise to distribute it gratis to the respondents to the Mathematics Questionnaire. I hope that you will immediately take any action open to you to further the implementation of those recommendations of the Study with which you are in agreement. I hope also that you will publicize the Report encouraging your friends and the organizations with which you are associated to order the definitive edition which is expected to appear in the fall of 1975 through Information Canada.

I am personally responsible for the main body of the text. However, the first two chapters are largely based on an excellent first draft by Dr. Gordon Edwards. Klaus Beltzner has been largely responsible for the Appendices. The main effort - and it was considerable - in designing and analyzing the Questionnaire was due to Klaus and Gordon. In the period July 1, 1973 to August 31, 1974, Dr. Edwards gave his full time and extraordinary energy to the day-to-day activities of the Study. I am most grateful to him and Klaus Beltzner for the contribution they made towards the completion of this Study.

I am also grateful to Dr. P.D. McTaggart-Cowan who, as Executive Director of the Science Council of Canada consistently gave the Study enthusiasm and counsel; to Professor Nobert Lacroix for his understanding of my failures and for the careful and objective way in which he chaired the Joint Committee; to all the members of that Committee whose cooperation made it all possible, and whose perceptive criticisms improved the Report; and to our secretaries Mrs. Lynn Tremblay, Kathy Brady and Maggie Adams who handled their mathematicians with equanimity and great efficiency.

We are aware that the Report contains no discussion of research publications nor of the special problems of women mathematicians. This is because there were no submissions to the Study on these topics. Doubtless, there are other grave omissions. We will be most grateful to readers of this Preliminary Edition who bring to our attention other serious lacunae, and, especially errors of fact. If the latter are communicated to us immediately they may be caught in the final edition. And, of course, we will welcome arguments about the recommendations of the Report. You may write to Professor Norbert Lacroix or the undersigned, in care of the Science Council of Canada, 150 Kent Street, Ottawa, Ontario, K1P 5P4.

A.J. Coleman

Introduction

"MATHEMATICS CANADA is not another bureaucracy. It is a study which will attempt to bring into sharp focus the present state and role of the Mathematical Sciences in Canada, and recommend policies for the future". (From a Press Release issued by the Science Council of Canada, September 1973.)

After some years of debate over science policy in Canada, specific plans for action are gradually emerging within the scientific community. This Study describes a debate on policy matters relating to the Mathematics Study and simultaneously makes several proposals for action. It shows the leadership of the mathematical community itself in assessing its present contribution and in trying to enlarge its role to serve the social cultural and economic needs of Canada.

To assume such leadership was not possible five years ago because the mathematical community was fragmented into several units each with its own concerns. The Study has brought them together on a common concern and constitutes the first step in improving things at the political level for the mathematical community, which has been ignored mainly because it has been divided, not visible and without a national voice. Other common concerns have since emerged, such as follow-up of the Study and implementation of proposals for action, to strengthen the mathematical community's desire to remain on stage.

The effect of the Study has been felt even before the appearance of the present study because the actual process of the Study made it a vehicle of change. The Study contains numerous indications of what is yet to be expected: gradual changes of attitudes within and without the mathematical community and promotion of meaningful communications at all levels.

Mathematics and the Mathematical Community

In the twentieth century, the mathematical community has become very diversified - encompassing not only teacherss and researchers, but also a good many of those who work in fields such as Statistics, Actuarial Sciences, Operations Research, and Computer Science. In short, it takes in all those who use mathematical thinking as an integral part of their job. The various fields mentioned above are collectively referred to as the Mathematical Sciences, and throughout the Study, words such as "mathematics", "mathematical", and "mathematician" are often used to refer to this entire spectrum of mathematical activity (as described in Chapter II, "The Mathematical Ecosystem").

It is perhaps not superfluous to point out that there is a distinction between mathematics and the mathematical community. One will hear such statements as "Mathematics will not be able to weather the stormy waters which she has entered without a profound change in her aims and orientation" (A. Zadeh, as quoted in Chapter II). In such cases, mathematics itself is really removed from the situation, standing as an independent body of knowledge as glorious as ever. It is the mathematical community which is being pointed at, because of its activities over the past thirty or forty years. It is this type of activity which is now under wide discussion.

Management of the Study

The Joint Committee and the Executive Committee, as outlined at the beginning of Chapter I, have supervised and managed in a general sense the conduct of the Mathematics Study. The Study Director, the Assistant Study Director and the Project Officer were responsible for the day-to-day work and initiatives. In addition, they have attended the seven meetings of the Joint Committee and the three meetings of the Executive Committee. The meetings took place in a fine spirit, characterized by many constructive suggestions and criticisms which emerged.

Professor C.F.A. Beaumont of the Faculty of Mathematics at the University of Waterloo has taken a very important part in the Mathematics Study, by devoting much thought, time and energy to the organization of a series of one-day seminars involving representatives from industry or government and university mathematicians. In particular, he made all the preparations for five of the eight seminars and chaired four of them himself.

Details pertaining to the process of the Study constitute Chapter I of the Study.

On the Main Body of the Study

The opinions and findings in the main body of the Study are those of the authors, K.P. Beltzner, A.J. Coleman and G.D.J. Edwards. The Joint Committee has authorized the publication of the Study in the hope that its stance on many important issues in Mathematics will trigger discussions, actions or reactions leading to improvements in Mathematics in Canada.

In particular, it is hoped by the Joint Committee that the Report and the recommendations will be considered at length by the sponsoring societies.

Position of the Joint Committee on the Recommendations

The Joint Committee supports the seven recommendations as they appear in Chapter XI. The vote on Recommendations 1, 2, 3, 5 and 7 was unanimous.

Recommendation 6 on the creation of a Canadian Institute of Applied Mathematics was carried with one abstention (that of Mr. J. Faille, there being agreement that his vote be reported). Professor P.G. Rooney, who could not attend the meeting at which Recommendation 6 was adopted, stated in writing prior to the meeting that he would have supported a weaker version proposing a study of the opportuneness of creating a Canadian Institute of Applied Mathematics.

Recommendation 4 on a review of granting policies of the Federal Government was adopted with one opposing vote by Professor I.B. MacNeill. Again it was agreed that his position be reported [1].

Although the councils of the sponsoring societies have not formally approved the recommendations, their representatives on the Joint Committee have endorsed them as indicated above and have agreed to bring them before their respective organisations.

In particular, the members of the Joint Committee will strongly recommend to their own socieites that they act quickly to create a continuing Council for the Mathematical Sciences (Recommendation 7) to further the cooperative work that began with sponsoring the Mathematics Study.

On behalf of the Joint Committee,

Norbert Lacroix
Chairman

10 July 1975

Footnote to the Introduction

1. Professor MacNeill has made the following statement: "I would have supported Recommendation 4 if it had also contained an additional section as follows:
   (d) The feasibility of a separate Grant Selection Committee being established for Probability; Stochastic Processes; and Statistics, both pure and applied".

Foreword

In writing this Foreword on the eve of my retirement from the Science Council as its Executive Director, I feel in the mood to go beyond the normal confines of Foreword writing and to make some rather personal comments on this study.

As one of many in the scientific community in Canada whose feet were firmly embedded in mathematics but who are now engaged in other pursuits, I viewed with disquiet, which turned to dismay, the growing post World War II fragmentation of mathematics in our universities and the onset of conflict which could only be described as civil war.

In the establishment of Simon Fraser University in the middle '60's, we attempted to do something about it, to start healing the wounds between the pure and applied mathematician and between mathematicians and scientists in other areas. The extent to which this was achieved initially, and to which it has flourished subsequently, is for others to comment on but the intent was there.

When I took on the job of Executive Director of the Science Council in September 1968, I sensed, as is brought out in this present essay, that the mood of mathematicians at some locations in Canada was starting to change. There seemed to be a growing realization that a continuation of the posturing of the past two decades was an open invitation for a collision with disaster. There seemed, therefore, to be an opportunity to help in reversing the trend and to get the various parts into which mathematics had divided itself on converging courses and to start healing the wounds between mathematics and the rest of human knowledge. I had a personal goal therefore that the Science Council, at the opportune time, should play the role of a catalyst in bringing this about.

Notwithstanding this desire, the opportunity was frustratingly slow in occurring and the idea was frequently set aside by the constant battle of higher priority areas demanding the Council's time and attention. Thus, mathematics missed out on the early round of "inventory studies" of science in Canada that were undertaken by the Science Council and therefore became subject to the Council's current view that "inventory studies" should be done by those who are in the discipline, with the Science Council being willing to finance the study, providing the necessary enthusiasm and determination could be found within the discipline to do a good job. It was late in 1971 before there seemed to be any significant stirrings among the mathematical community. Finally, in the spring of 1973, the major segments of the mathematical sciences formed their consortium and put forward a good proposal for a study which led the Science Council to agree to fund the project.

I have been delighted with the way the six collaborating societies have not only come together but stayed together and tackled the job with determination and enthusiasm; the consortium's Chairman, Professor N.H. Lacroix of Laval University, can be justifiably proud of what has been achieved. Their actions have given the lie to many of the "Doubting Thomases" who were standing on the sidelines waiting for the consortium to blow up.

The Joint Committee for the Mathematics Study was most fortunate in being able to attract Dr. Coleman to be Study Director. Among Canada's leading mathematicians, he was uniquely qualified.

The consortium and the Science Council were both fortunate that Queen's University recognized the importance of the Study and generously released Dr. Coleman from duties and responsibilities to the University which, in other circumstances they would have a right to expect.

While Dr. Coleman has been completely frank in his essay on its limitations and shortcomings, it must be borne in mind that this kind of science policy investigation, representing a combination of survey, of analyses, of synthesis, of attitude and opinion sampling, coupled in the final stages with raw but hopefully informed judgement, is fraught with all the difficulties of other types of research. None of the studies undertaken under the aegis of the Science Council while I have been its Executive Director have really met all of my initial expectations or been completed within the originally conceived timetable. This probably means that we are incurable optimists, continually striving to grasp beyond our reach. So be it. In my judgement, the sponsors of the present study on mathematics need apologize to no man. It provides a platform upon which hopefully other institutions will find it in their interest to build. Still others must pick up the torch that has been lighted, carry it the length and breadth of Canada, and light fires under a sufficient number of people that the actions that have been identified as urgent are undertaken.

While I am enjoined to declare in every Foreword to an authored study that the opinions expressed do not necessarily reflect the views and opinions of the Science Council, I must add in this case that I hope it does.

I think it is important for all of us to regard the publishing of this essay as the beginning and not the end of change. Follow-up both with the community of mathematical scientists and those called upon to teach mathematics in the school system and follow-up with government and industry must take place. Discussion must continue until action is assured.

To this end, I think it is essential that the societies drawn together in the consortium for the purpose of this study form a Council to continue the cooperative effort now started, orchestrate the symphony of change and on occasion seize the baton and become the conductor.

I think the chapter on the teaching of mathematics in the university is critical as indeed is its companion chapter on research. There should be no such concept as a surplus of PhD's in mathematics. If indeed we can recapture the central thrust of a university degree being to teach people how to think, surely someone who has been taught to think at the highest level in the field of mathematics should be the most ubiquitous of scholars with a mobility to carry these talents into any area of human knowledge and function with distinction. This becomes of central importance as we see all around us and particularly at the political level the denigration of merit and the flight from the authority of knowledge.

It is the narrow and bigoted view of the mathematics doctorate clearly condemned in Coleman's essay which has put blinkers on good students and restricted their post-degree mobility. Once this is removed, Canada will need all the PhD mathematicians that the universities can produce. They will be few enough in number but they will be found thinking their way through all manner of problems seemingly remote from mathematics but only to the uninitiated.

I think the role foreseen for those with an M.A. in one of the mathematicial sciences is probably greater than we have the courage to articulate. If we are to "Mathematize with Joy" throughout the country - the journeyman evalgelist in industry and government may well be the "man with the Masters" (he or she).

I must admit my bias in stressing the need for urgent mathematical surgery through the medium of the Canadian Institute of Applied Mathematics in the whole field of survey methodology. The amount of money we are wasting in Canada, let alone the time and energy, in meaningless statistical compilations that nobody uses or should use, the demands on citizens to complete stupid questionnaires that are without serious statistical foundation are legion. The need for highly focussed statistics in the planning of Canada's future is undeniable but the way we are going about it is a travesty of logical thought; a mental blindness that would defy even the genius of a George Boole to comprehend. We can do so much better if we have a mind to do so.

In this day and age, when led by the federal government the country seems determined to deny merit and renounce the authority of knowledge, it is a pleasure to read the work of one who has the courage to uphold the importance of both knowledge and merit and to do so with the enthusiastic use of good English.

Perhaps it is John Coleman himself who shines through these pages that make them far more than a recipe for mathematicians or perhaps it is a combination of this and the mathematical rigour which is his creed. Whatever the reason, the result is a charming essay of great importance. It should be read by all.

P.D. McTaggart-Cowan
Executive Director
17 July 1975

Mathematics Study

Members of the Joint Committee

The six sponsoring societies and the Science Council of Canada are each identified with their representatives on the Joint Committee. The six members who also served on the Executive Committee are designated by the asterisk which precedes their name.

Science Council of Canada

(*) Dr. P.D. McTaggart-Cowan (till May 31, 1975)
Executive Director

Mr. J.J. Shepherd (starting June 1, 1975)
Executive Director

Mr. F.J.L. Miller (till August 1973)
Project Officer

Dr. R.P. Charbonnier (till December 1973)
Project Officer

Mr. Klaus Beltzner (starting January 1974)
Project Officer

Mr. J. Basuk (starting September 1973)
Secretary

Canadian Mathematical Congress

Professor A.H. Lachlan
Simon Fraser University

(*) Professor N.H. Lacroix (Chairman)
Universite Laval

Professor P. Lancaster
University of Calgary

Professor P.G. Rooney
University of Toronto

Canadian Operational Research Society

Dr. N.J. Hopkins
Department of National Defence

(*) Mr. E.L. Leese
Department of National Defence

Canadian Institute of Actuaries

Mr. J. Faille
Universite Laval

(*) Mr. R.M. Hammond
Department of Insurance (Ottawa)

Canadian Information Processing Society

Dr. F. Fiala
Carleton University

(*) Professsor H.S. Heaps
Concordia University

Statistical Science Association of Canada

Professor I.B. MacNeill
University of Western Ontario

American Statistical Association, District 7

(*) Mr. O. Tomasek
Bell Canada

Chapter I

The Study as Process

> *"Surprisingly little attention has been given to the Mathematical Sciences in Canada, although the need for long-range policy planning in this area seems obvious. The present study will be breaking new ground in attempting to provide an overall view of the role of mathematics in Canadian science, education, government and industry. In fact, the actual process of the study may be of greater importance than any written report which will result [1]."*

Within the last few years, mathematicians in Canadian universities have become increasingly conscious of the need to thoroughly reassess our values and goals. We have been moved because our performance in "service" courses is under increasingly perceptive criticism, many of our ablest doctoral students are failing to find satisfying employment, budgets are getting very tight, and enrolment in honours mathematics programs is on the decline in many universities in North America. We are also disturbed by more subtle reasons associated with a vague feeling of aimlessness and a growing desire to communicate with others. This mood is especially common among our younger colleagues. Many mathematicians have an acute sense of loneliness which issues from our inability to relate our work to the rest of life or to explain it to anyone but a few fellow mathematicians.

Against the background of such a mood several Canadian mathematicians in the late 1960's concluded that the moment was opportune for a study of the role of mathematics in our society. They quickly realized that for such a Study to be meaningful it could not be restricted to the work and interest of academic mathematicians nor merely to "mathematics" in the narrow sense.

In 1971, the Canadian Mathematical Congress established a special committee [2] on Policy for the Mathematical Sciences in Canada. A non-event which helped trigger this action was the failure of mathematicians to provide any organized input to the Senate Special Committee on Science Policy chaired by the Honourable Maurice Lamontagne - a failure which demonstrated quite dramatically the lack of cohesion and self-awareness of the Canadian mathematical community. Concurrently, and quite independently of the CMC, the Science

Council of Canada had begun to consider the advisability of launching a critical scrutiny of the role of the Mathematical Sciences in Canadian society. These two streams merged. After consulting with a representative of the Science Council [3], the CMC Committee proceeded to enlist the cooperation of five other Canadian mathematical societies, to mount a Background Study on Policies for the Mathematical Sciences in Canada on behalf of the Science Council of Canada.

A Joint Committee of fourteen members was formed, representing the six sponsoring mathematical organizations and the Science Council of Canada. At their first meeting, the members of the Joint Committee unanimously agreed that a study of the Mathematical Sciences in Canada was long overdue, and that cooperative efforts between all workers in the mathematical sciences was desirable and much needed. An Executive Committee of five members was formed, a Chairman elected, a Study Director named, and an Assistant Study Director chosen. In the spring of 1973, the Joint Committee contracted with the Science Council of Canada to undertake a study of the Mathematical Sciences in Canada, using offices in Ottawa provided by the Science Council.

Initial Steps

As spelled out in the contract, the objectives of the Study were conceived in rather ambitious - almost grandiose - terms:

"1. To establish what kinds of mathematics are currently used in Canada, and to what extent. Also to suggest areas of mathematics which are not being studied or used to the extent which would be desirable.

2. To describe and evaluate various possible types of research in the mathematical sciences.

3. To estimate present and future manpower supplies and needs at various levels of mathematical competence.

4. To examine current objectives and methods of training in mathematical activities and to suggest possible improvements at all levels.

5. To recommend methods by which significant real problem areas amenable to mathematical treatment can be identified and publicized on a systematic and continuing basis."

With hindsight it is now apparent that these objectives were well-nigh impossible to attain in any definitive sense, certainly not with the resources at the disposal of the Study. Nonetheless, the stated objectives were frequently discussed in the Joint Committee and pondered by the Director of the Study and his Assistant. They had an invaluable steering effect on the study process.

Despite the apparent emphasis on data-gathering in the statement of objectives, the Mathematics Study was never conceived as a mere inventory study. From the first it was concerned with long-term strategies for the development and utilization of the Mathematical Sciences in Canada. This is in keeping with the mandate expressed in the Science Council of Canada Act of 1969, which makes it clear that the Council's primary duty is to assist in the formulation of science policy in the national interest.

The Science Council had sponsored previous studies [4] concerning physics, chemistry, biology and the earth sciences. On the basis of his association with these studies, Dr. P.D. McTaggart-Cowan - the then Executive Director of the Science Council of Canada - insisted that the most important effect to aim for was a change of attitude - by teachers, by university professors, by managers in business and industry, by government officials, and by parents. The most comprehensive collection of facts about the mathematical sciences in Canada, the most telling diagnosis of the state of mathematics or the most brilliant set of recommendations would be of no avail unless the Study aroused interest and debate. If changes are necessary, they can be brought into effect only if they win the approval and support of those who are most directly affected - the people who are expected to adopt and implement the new ideas.

Three guiding principles emerged during the first few months:

(i) the universe of mathematical activity, from the use of simple arithmetic in our common life to the application of statistics and computers or abstract research on category theory, consists of a complex interaction of many different species of mathematicians connected by a communication network - a veritable <u>ecosystem</u> as we shall refer to it in Chapter II;

(ii) the study should not be restricted to a passive role, merely observing the system from the outside as if it were a static thing. Rather the process of the study should itself be a vehicle of change. This meant opening up new channels of communications and stimulating a vigorous debate among all those who cared enough to express their views about the future of mathematics in Canada;

(iii) the study should start not with the assumption that all of the important issues were necessarily known in advance, but use the study leaders as coordinators of a national effort to identify the true issues and to make effective recommendations.

It was felt that these three guiding principles would help to prevent the Study from drawing a distorted picture or proposing unworkable solutions. The success of the Study therefore depended upon the active cooperation and involvement of every part of the mathematical community.

The Actual Process

To our knowledge, there has never previously been an attempt to delineate the total role of mathematics in a modern technological society. The Directors of the Study therefore had to break new ground and no one will be more aware than they are of its many inadequacies. In order to gain insight into the actual situation and to obtain some idea of how the creators and users of mathematics viewed the mathematical ecosystem, four techniques were used: briefs were called for; eight one-day Seminars were held; Universities, Industry and Government Agencies were visited; a Questionnaire was circulated widely.

(i) Briefs

In order to publicize the study, a press release was prepared and distributed to newspapers, radio stations, professional and scientific and technical societies, and mathematics teachers' associations within the various provinces. This press release gave quite a detailed account of the Mathematics Study, and invited submissions from any individuals or groups who cared to express their views on the role of the Mathematical Sciences in Canada today. In particular, professional societies and teachers' associations were urged to publicize the study among their members in order to make sure that no one would be denied the opportunity of learning about the study and participating in it. A small but significant number of letters and briefs were received as a result of this initiative, mostly from teachers and educators.

We asked university deans to furnish the names of people in various departments who were particularly concerned with the impact of the Mathematical Sciences in their disciplines. These individuals were then contacted by letter and asked to express their views in response to the five following questions:

(a) What kinds of mathematics are currently being used in your discipline, and how are they being used (at the undergraduate, graduate and research levels)?

(b) To what extent is mathematics likely to influence your discipline in the future, and in what general direction?

(c) Are students in your discipline currently acquiring the necessary mathematical tools, skills, and attitudes? Are there any notable omissions of content or inadequacies of approach in mathematics education? (Some indication of "core content" and "key skills" would be helpful.)

(d) What practical mechanisms can you suggest to facilitate cooperation with mathematicians in matters of curriculum design, joint research, interdisciplinary seminars, etc.? To what extent are these desirable goals?

(e) What are the chief obstacles (administrative, financial, academic, professional, etc.) preventing fuller cooperation between mathematicians and users of mathematics? Any suggestions for improvements?

The response to this initiative was very gratifying, resulting in about a hundred briefs and letters from physicists, chemists, engineers, economists, biologists, psychologists, educationalists, linguists, agriculturalists, medical researchers, and so forth. Almost without exception, these submissions reflected a deep concern for the subject which went far beyond the preceding five questions. There was near unanimity that mathematics could be used more wisely and more effectively if there were closer ties between mathematicians and users of mathematics. Time and time again, the vital importance of attitudes was emphasized: attitudes on the part of students, on the part of mathematicians, and on the part of users of mathematics. As one correspondent said, "Without a change in attitude, it is difficult to see how progress can be made. Current attitudes that scientists have of mathematicians need to be changed. A genuine interest on the part of mathematicians for the problems of science would help to change them."

(ii) Eight Seminars

A series of one-day seminars was also organized to bring together mathematicians and managers from universities, community colleges, business, industry, and government, in order to discuss matters of common concern. Two of these seminars were of a general nature, one was focussed on the transportation industry, another dealt with mathematics in the resource industries, while still another concentrated on financial institutions and management consulting firms. The remaining three were devoted to the role of the Mathematical Sciences in the Federal Government, under the following three headings: Mathematics and Policy Planning, Mathematics Statistics and the Environment, Mathematics and Technology. At each seminar, the participants were divided evenly between academics and non-academics, and the discussion was directed to the following topics:

(a) the suitability of post-secondary mathematical education for purposes of non-academic employment;

(b) personal qualities and professional qualifications which are sought after in the hiring of mathematical scientists for non-academic work;

(c) present and future manpower needs in various areas of mathematical expertise;

(d) the desirability and feasibility of fostering interchanges of personnel or joint projects between Departments of Mathematical Science and mathematically intensive organizations;

(e) prospects for continuing education in college-level mathematics for managers and for mathematical practitioners.

In order to make these seminars as instructive and as relaxed as possible, each contingent was asked to prepare a written brief in advance which could be distributed to all the other participants. This took a good deal of the pressure off the meetings themselves, and the resulting briefs (some of which were surprisingly detailed and ambitious) were included as part of the proceedings which were circulated for each of the eight seminars.

The proceedings of the three Ottawa Seminars were edited by Dr. Gordon Edwards and were issued by the Science Council of Canada in a limited edition under the title Mathematics in Today's World (XXV 360 pp.). This report exhibits the kind of mathematics and their applications currently employed by the Federal Government. Copies of the report were deposited in libraries of the Federal Government and Canadian universities. The proceedings of the other five Seminars were edited by C.F.A. Beaumont and distributed privately. Their contents are summarized in Chapter III.

Those who attended these events found them informative and thought-provoking. Many people hailed them as marking the beginnings of a dialogue that should have begun long before, and suggested that such meetings should be continued on a more regular basis. (Since they were mainly organized locally and hosted either by a university or by industry, it should be possible to replicate these seminars manyfold by local initiative.) Despite great differences in detail, the same fundamental points were raised independently in one seminar after another: the use of mathematics is definitely on the rise, and the main obstacle to its more effective use can be found in the attitudes which people have inherited from the past. Managers are not used to taking a scientific approach to problem solving, and they will often sacrifice long-term gains for the sake of short-term expediency by not providing a proper atmosphere for research and development. Mathematics graduates are not used to coping with unstructured problems, working with messy data, obtaining approximate solutions, and communicating with people who are mathematically illiterate, which greatly hampers their usefulness on the job. Mathematics teachers have little or no experience in teaching the kind of mathematical thinking that is needed for coping with problems which arise in business, government and industry, and this prevents them from preparing their students for the possibility of non-academic employment. Employers of mathematicians are often too impatient to wait for results because they do not understand the difference between a scientist and a technician; as a result, they may be unwilling to take the time and effort to provide thorough on-the-job training as a sound investment in the future.

Such habits of thought are hard to change, but as long as there is a dialogue taking place, the prospects are hopeful. The real significance of the industry seminars held in conjunction with the Mathematics Study is that people on all sides have proven that they are willing to engage in such a dialogue. Having established that much, the problem now is to keep the dialogue going.

(iii) Visits

During the fall of 1973, the study leaders visited 24 university campuses in Canada in order to explain the aims and objectives of the study to university mathematicians and users of mathematics. The importance of their involvement in the study was stressed, and they were urged to set up discussion groups for the purpose of producing joint briefs on particular topics of concern to the study. Because of the general lack of communication within the mathematical community, it was hoped that these discussion groups would help to bring about a more realistic and balanced overview of the current situation. If a problem is viewed from several different perspectives simultaneously, it sometimes happens that spurious impressions and simplistic approaches will be more easily identified and discarded. Several discussion groups did materialize, resulting in a good number of substantial, well-reasoned briefs on the present and future role of mathematics within the academic community. In addition, a large number of individual submissions were received from university mathematicians on many different topics, including the undergraduate teaching of mathematics, the dangers of over-specialization, the need for diversification of mathematical activities, the plight of mathematics graduates who are unable to find employment, and possible mechanisms for change. Virtually all of these briefs accepted the need for university mathematicians to become more involved in the applications of mathematics, and many of them expressed profound concern over the isolationism which was the de facto policy of most Mathematics Departments in the recent past. Judging from the views expressed in these briefs and in the industry seminars, it is quite clear that there has been a substantial change of attitude among university mathematicians in the last few years. Many pure mathematicians who have no particular interest in applications would welcome alternative policies, provided that their own freedom to pursue mathematical research for its own sake is not seriously impaired.

Visits were also paid to mathematically-intensive organizations in various regions of Canada, and interviews were conducted with top administrators and mathematicians operating in a supervisory capacity. The most striking feature of these interviews was the widespread belief that mathematics graduates are not necessarily the best candidates for non-academic mathematical employment. There is a growing demand for people with mathematical skills, but most employers seem to prefer hiring people who have a background in some other discipline and who have picked up their mathematical knowledge along the way. Additional visits and interviews were conducted at high schools, community colleges, boards of eduation, and provincial Departments of Education.

(iv) Questionnaire

A five-page mathematics questionnaire was sent to all graduates from Canadian Universities in the Mathematical Sciences in certain selected years. As a means of further publicizing the study and obtaining comparative information, the questionnaire was also mailed to members of the six sponsorsing societies. This questionnaire, which is reproduced in Appendix I, was a main data-gathering device used by the Mathematics Study. There were 3,231 usable returns. Of these, 1,830 were from Canadian university graduates, representing a 29.6% response rate. The remaining 1,401 resulted from the mailing to members of the sponsoring organizations, but because of overlapping memberships (one individual received 5 copies of the questionnaire), the response rate cannot be estimated with any accuracy but was at least 20%. These response rates may be compared with the typical 5%-10% characteristic of commercial market surveys and 35%-80% common for Statistics Canada surveys for which the citizens questioned are legally obligated to respond.

The Study had neither the resources nor the time to make a definitive statistical enquiry into the role of the mathematical sciences in Canada. We strongly warn readers to avoid making large and sweeping generalizations solely on the basis of the results of the Questionnaire reported in Appendix I. The Mathematics Study Questionnaire should be regarded as a pilot-study on the basis of which it may be decided that more specific and statistically sounder future surveys are called for. Even so, these results are quite striking in certain respects and will be of interest to anyone seriously concerned with the teaching and/or use of mathematics in Canada. They represent a first - and therefore in itself interesting - reconnoitre of the Canadian mathematical "universe".

When one considers that the direct costs to the Study of printing and distributing the questionnaire and analyzing the responses was less than $7,000, it is clear that the effort was most worthwhile. It gives us the opinions of 1,830 graduates of Canadian mathematics programmes about the education they received. It also provides a sampling of the wide variety of places in Canadian society where persons with mathematical training are to be found.

Though we cannot be certain that the 1,830 respondents constitute a random cross-section of all graduates of the mathematics programmes of Canadian universities, there are two pieces of evidence suggesting that they are. As can be seen in Appendix I, the distribution of opinions over respondents from different graduating years appears to be remarkably stable. This provides some internal evidence of the reliability of the results. However, much more significant, in our opinion, is that the views expressed by the respondents to the questionnaire agreed, on the whole, with views that were pressed upon us again and again in Seminars, briefs and letters.

Estimating Manpower Needs

Less than one month after the Study got underway, we recognized that it would be impossible to achieve the original third objective of the Study in any meaningful quantitative sense. To estimate the supply and demand for mathematical manpower requires a satisfactory definition of the term "mathematician" in a non-academic context. Is every statistician to be regarded as a mathematician, even though his work may involve nothing more than routine regression analysis? If not, then where do you draw the line? How do you determine which computer programmers are using a significant amount of mathematics in their jobs? On what basis do you decide whether or not a scientist who is engaged in mathematical model-building should be called a mathematician? Should mathematicians be defined in terms of their education or their work, or both? These difficulties, which are formidable enough by themselves, are compounded by the fact that companies and government departments do not usually classify their employees as "mathematicians", even if they are doing work of a highly mathematical nature. Job classifications are more often based on the kind of problems which arise than on the tools that are used to solve them. The result is that most large organizations, even if they are mathematically intensive, are not really in a position to say how many mathematicians they have working for them. Figures which are supposed to represent the number of mathematical employees are often very misleading, because they are based on arbitrary labels which may not fit the situation well at all.

The Public Service Commission of the Federal Government is a case in point. They have introduced a new Mathematics Classification which is supposed to include all employees of the federal government who are concerned with developing and applying mathematical methods at a suitable level of sophistication. As of October 1, 1973, there were exactly 109 employees in this category, almost all of whom were statisticians. Nevertheless, a computer search of the DATASTREAM records maintained by the Public Service Commission revealed the following facts (as of September 18, 1973):

1. 1,526 federal employees indicated that they have at least a master's degree in mathematics.

2. 1,306 federal employees used the word mathematics in describing their past and/or present field of work.

3. 328 federal employees indicated that they have at least a master's degree in mathematics <u>and</u> used the word mathematics in describing their past and/or present field of work.[5]

DATASTREAM does not cover all federal employees and subsequent investigations of selected government departments have succeeded in confirming our suspicion that some of the most significant pockets of mathematical activity in the federal government are not yet represented in the new Mathematics Classification. In fact, one PhD mathematician in the federal government (who is not classified as a mathematician) confided that most people in his position would rather not be labelled as a "mathematician", for reasons of prestige and salary.

Another cautionary experience was obtained when Bell Northern Research was asked to supply information concerning its mathematical employees. In the course of a two-hour interview, the study leaders were told that "Mathematics is very important at BNR, but mathematicians, as such, are not. We have not had much luck with mathematicians in general; the real successes are engineers who have done some graduate work in mathematics." In this context, the term "mathematics" clearly refers to the person's educational background in mathematics. However, the BNR University Liaison Officer, Robin R. Jackson, kindly undertook to prepare a list of employees at BNR who are actively engaged in work of a highly mathematical nature, including all their university degrees and fields of specialization. It was then discovered that 76 out of 126 had degrees in mathematics (excluding degrees in computer science). Two of these had <u>doctorates</u> in mathematics, 26 had <u>master's</u> degrees in mathematics, and the remaining 48 had <u>bachelor's</u> degrees in mathematics (frequently accompanied by a higher degree in another discipline). Mr. Jackson confessed that he was quite surprised to discover how many mathematics graduates there were at Bell Northern Research. It may well be that the enviable world-wide reputation enjoyed by BNR for the sophistication of its research and development is largely due to the mathematical competence of its staff.

These experiences, which are not atypical, convinced us that questions addressed to personnel officers concerning the number of mathematical employees within their organizations would likely meet with very erratic responses. The problem is that there is no accepted definition of what a mathematician (or a mathematical scientist) should be - not only among managers and executives, but even among mathematicians themselves. For instance, an applied statistician may actually resent being called a "mathematical statistician", because that term suggests to him a certain esoteric and highly theoretical brand of statistics which is quite unlike the practical work which he does. Indeed, many practically oriented mathematical scientists are extremely reluctant to allow themselves to be described as mathematicians. Apparently, the word "mathematican" has become associated in people's minds with the academic, the theoretical, or the incomprehensible.

But even if the term "mathematician" were scrupulously defined in advance, and carefully purged of any ivory tower connotations, the personnel office would still be hard-pressed to interpret the definition correctly in each individual case. Personnel records are not designed to separate mathematicians from non-mathematicians. Attempts to obtain manpower information of this kind from several branches of the federal government demanded an inordinate effort from the individuals involved.

Because of these vexing problems of definition and interpretation, all of the existing data concerning the non-academic employment of mathematicians in Canada is of very questionable validity. For example, in the 1971 Canadian Census, mathematical employees in government and industry could have shown up in any one of three groupings[6]:

1. Mathematicians, Statisticians, and Actuaries.

2. Systems Analysis, Computer Programming, and Related Occupations.

3. Occupations in Mathematical Statistics, Systems Analysis and Related Field, (not elsewhere classified).

These categories are not exclusive, not exhaustive, and not homogeneous in composition. Further, since the respondents are asked to describe their work in their own words, it depends entirely upon clerical interpretation whether or not a given mathematical employee will eventually end up in one of the three categories specified above. In the final edition we will hopefully be able to present some of the results[7] related to the Mathematical Sciences of the Highly Qualified Manpower Post Censal Survey (1973) conducted by Statistics Canada on behalf of the Ministry of State for Science and Technology. In particular, we expect to illustrate the kinds of problems encountered in the interpretation of the results related to the non-academic mathematical occupations.

Attitudes in Process

During the course of the Study we were both surprised and greatly heartened by the cooperation we experienced and the interest with which we were received on nearly every hand. We quickly gained the feeling that everyone in Canada regarded the Mathematics Study as most opportune: parents struggling with the "New Maths"; managers in industry and government coping with the computer revolution; school teachers concerned about the choice of curriculum and teaching methods; mathematics professors wondering how to improve their service courses, or distressed that their PhD students were unable to find positions; workers in the physical, life and social sciences who know that they need more powerful mathematical tools but find formidable obstacles in communicating with academic mathematicians. Of course, nearly all engineers are convinced that they can teach mathematics better than any mathematics professor! We met a farmer who has taken his nine-year old son out of school in order to teach the boy mathematics himself since he regards the inculcation of a love for and command of mathematics as the absolutely crucial element in a sound education. And, as we had anticipated, every mathematics graduate was eager to tell us how his undergraduate program could have been vastly improved!

Many pure mathematicians who specialize in the more esoteric branches of their subject feel somewhat threatened today for a variety of reasons. Because of the auspices of the present Study, we rather expected to be greeted with suspicion - even hostility - in Mathematics Departments. To his surprise, the Director of the Study encountered a mere suggestion of such an attitude in only two of the twenty-one Departments he visited. Perhaps this Report will evoke a stronger reaction.

The literary deposit of the Study is approximately 300 letters and briefs, reports of the eight Seminars and eleven computer tapes on which[8] have been encoded the results of the Questionnaire. These contain much more information than we can hope to report in the following chapters. We shall be able merely to sketch the highlights of the huge input to the Study. At the same time, we shall formulate the conclusions to which we have been led concerning the chief obstacles and opportunities which face all of those who are concerned that the mathematical sciences should flourish in Canada and contribute effectively to the common good.

We gladly anticipate that our opinions and conclusions will be challenged. A vigorous responsible debate will further the process of mathematical-consciousness-raising to which we believe the Study has already given some little impetus.

References to Chapter I

1. Press Release, Science Council of Canada, August, 1973.

2. The Committee consisted of R. Adams (University of British Columbia), C.F.A. Beaumont (University of Waterloo), A.J. Coleman (Chairman; Queen's University at Kingston), G. Sabidussi (Université de Montreal).

3. Mr. Fred Miller was the representative of the Science Council who, in collaboration with Dr. Pierre Charbonnier, in June 1972, prepared a feasibility report for a study of the Mathematical Sciences. Mr. Miller of COMINCO (one of the world's leading producers of lead, zinc, silver, fertilizers, etc., and also of fabricated metals), and Dr. Charbonnier of the Federal Department of Energy, Mines and Resources, had both been seconded to the staff of the Science Council to work on background studies which were subsequently released under the title Canada's Energy Opportunities.

4. The Science Council Report No. 17 was entitled Lifelines: Some Policies for Basic Biology in Canada, (1972); Background Study

No. 2, Physics in Canada: Survey and Outlook, was written by a Study Group of the Canadian Association of Physicists headed by D.C. Rose (1967); Study No. 9, Chemistry and Chemical Engineering: A Survey of Research and Development in Canada, was written by a Study Group of the Chemical Institute of Canada (1969); Study No. 13 dealt with Earth Sciences Serving the Nation (1971); and Study No. 18 entitled From Formalin to Fortran: Basic Biology in Canada was written by P.A. Larkin and W.J.D. Stephen (1971).

5. The Mathematics Study is indebted to Doug Pearce of the Public Service Commission for making this information available.

6. Occupational Classification Manual, Census of Canada, 1971, Vol. I; which is based on the Canadian Classification and Dictionary of Occupations, Information Canada, Ottawa.

7. The H.Q.M. results have been tabulated by Statistics Canada in January 1975, but the Ministry of State for Science and Technology still awaits an acceptable Reliability Report before it will authorize public release. It is hoped that the data will be available in time for the final edition.

8. All this information is in the custody of the Science Council of Canada and is available for study by responsible persons. It provides a considerable body of research material suitable for Master's or Doctor's theses in sociology or education. The results of the Questionnaire are coded for processing by SPSS.

Chapter II

The Mathematical Ecosystem

"Euclid and Archimedes were undoubtedly supreme thinkers, and mathematicians have been able to reach further only because, as Newton said, they stood on the shoulders of such giants. Nevertheless, it is in our age that mathematics has attained its range and its extraordinary applicability. Present day Western Civilization is distinguished from any other known to history by the extent to which mathematics has influenced contemporary life and thought." - C.F.A. Beaumont[1]

In this chapter, we attempt to place the current mathematics situation in historical perspective, to depict the crisis which confronts the mathematical sciences in Canada, to draw an analogy between a complex ecosystem and the varied branches and outposts of mathematics, and to announce the principal themes of the subsequent development.

Mathematics Exploding

Isaac Newton invented the Calculus in 1665. Modern history began!

The reader may protest that Newton's discovery was only possible because of the work of Wallis and Barrow, of Euclid and Pythagoras. True. He might also argue that the modern world was the result of the Renaissance in Italy, or that it was shaped by a wide variety of political and economical forces. Many such considerations could be advanced with great cogency. Yet Alexander Pope's famous couplet asserts

"Nature and nature's laws lay hid in night:
God said, Let Newton be! and all was light."

It was Newton's mathematical explanation of the motion of the planets which seized men's imagination and unleashed the development of what we now refer to as "Newtonian mechanics" which was basic for the creation of modern technology leading, in our age, to air travel and space-ships. The Calculus of Newton and Leibniz, extended and refined by Gauss, Stokes and many others, was used by Maxwell to put electricity and magnetism on the firm theoretical basis which made possible the advent of the electric age: radio, TV, radar, high-power transmission lines and electronic computers.

Before 1900 only a few scientists and engineers were conscious of the key role which mathematics was playing in the technological process of Western society. The only mathematics visible in everyday life was the arithmetic of finance and the geometry of design. The role of mathematics in business, in industry, in government, in the biological sciences or in the social sciences, could have been understood by anyone with a good grounding in the basic skills of addition, subtraction, multiplication, division, and measurement. Only a handful of technical specialists - engineers, actuaries, physicists, and mathematicians - felt the need for more advanced mathematical concepts and methods.

Today the situation is very different. There has been an expansion of mathematical activity during the last forty years, affecting an extraordinary range of human activities. These include economic policy planning, manufacturing and advertising, urban planning, agricultural and medical research, geological exploration, the behavioural sciences, genetics, and ecology - not to mention such unlikely fields as anthropology, archaeology, and linguistics! The influence of mathematics has never been so pervasive or so subtle as it is today.

Most of the major theories of physics- particle mechanics, fluid dynamics, electricity and magnetism, elasticity and plasticity, relativity theory, quantum mechanics, and many others - have developed so far along mathematical lines that they have acquired a mathematical character all their own. These branches of theoretical physics, taken together in their present mathematical formulations, comprise what is known as Classical Applied Mathematics or Mathematical Physics. Twentieth-century engineering developments have succeeded in translating some of these sophisticated mathematical-physical theories into technological realities, including our present-day communications networks, nuclear weapons, aerospace technology, and electronic computers. Meanwhile, many of the methods and insights of Classical Applied Mathematics find unexpected applications in new fields such as mathematical economics, biophysics and geophysics.

However, in the present century, particularly since the last[2] war, there has been rapid development of new mathematical methods suggested by such terms as Information Theory, Systems Analysis, Optimization, Decision Theory, Algorithmics, and Control Theory. Pervading all these new mathematical disciplines and their application is a widespread use of statistics, mathematical modelling and computers.

Statistics is a mathematical science which draws heavily on the theory of probability (the mathematics of chance events) and linear algebra. It is mainly concerned with (a) collecting and processing data about large populations; (b) drawing inferences about an entire population based on the examination of carefully selected samples; (c) providing decision-makers with a scientifically-based analysis of the relative probability of various possible outcomes of a situation which involves uncertainty. In recent times, statistical techniques of experimental design and significance-testing have come to play a vital role in almost all of the experimental sciences, especially in the biomedical and social sciences. A noteable example is provided by the carefully designed experiments in plant-breeding which enabled the Canadian Department of Agriculture to produce superior strains of wheat particularly suited to the Canadian climate. Statistical procedures are now used to control the quality of manufactured products, design surveys and interpret their results, study the effects of pollutants on public health, predict changes in traffic patterns, or estimate economic parameters for purposes of forecasting. Such applications of statistical theory have become important primarily in the last thirty-five years.

Mathematical modelling is more of an art than a science. Its object is to represent the behaviour of a real-world situation by means of a carefully contrived mathematical description, usually referred to as a mathematical model. Such models may be simple or complicated, quantitative or qualitative in nature; either deterministic or probabilistic (in which case they are called "stochastic models"). They may use nothing more than simple algebra, or they may involve the most sophisticated mathematical concepts. The modelling approach has dominated the development of theoretical physics from the time of Newton to the present day. Only in recent times has mathematical modelling been applied to problems arising in management, in the life sciences, and in the study of human behaviour. The field of operations research, which came into being during the Second World War, makes extensive use of mathematical modelling techniques in order to aid managers to solve problems of decision and control, particularly those involving the movement and allocation of goods and services within large systems.

The usefulness and importance of both statistics and mathematical modelling has been greatly enhanced by the electronic computer, which has had an enormous impact on many facets of modern society. Large tracts of pure mathematics which previously were only of theoretical interest have suddenly become of great practical serviceability because of the existence of computers. Using the computer's powers of data storage and manipulation, a mathematical approach has been taken to thousands of problems which simply could not have been dealt with previously. How could the businessman think effectively about the best method for inventory control, when he did not even know (and could not even guess) how the inventory was actually changing from day to day? How could the biologist study the effects of environmental changes on a complicated ecological system composed of several interacting species, when neither the variations in the environmental factors nor the fluctuations in population levels could be computed? Who could have imagined that weather forecasting would be achieved through a detailed study of the entire atmospheric fluid on a global scale, when the capacity for carrying out such a project was completely unthinkable? These are just a few examples of practical problems which have suddenly become susceptible to mathematical treatment, thanks to the computer.

The Core

This extraordinary development of the mathematical sciences has emerged from the central body of abstract thought traditionally referred to as "pure mathematics". In the twentieth century this central core of abstract mathematics has been growing very rapidly. The men and women who are forcing back the frontiers of pure mathematics today are exploring the wonders of mathematical structures that no one dreamed about a hundred years ago. Such mathematical researchers are usually far removed from the world of practical applications, absorbed in the contemplation of abstract mathematical systems of great beauty and subtlety. To such people, mathematics is an evolving art form whose medium of expresion is not words or sound or pigments, but thoughts. Results in pure mathematics are not judged by their utility, which is usually unknown, but by criteria of logical consistency and overall craftsmanship. Even if some of them are ultimately "useless", they will surely find a place alongside our most valued cultural possessions. Yet history has taught us the truth of A.N. Whitehead's famous judgement "The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact." [3]

Indeed, it is striking that many of the practical mathematical tools of today were first developed without reference to possible uses outside of mathematics. The English mathematician, Cayley, firmly believed that matrices, those rectangular blocks of numbers which he studied in the mid-nineteenth century, would never be applied to anything useful. They are now an everyday working tool of engineers, physicists, economists, statisticians and behavioural scientists. Complex numbers, involving the "imaginary" square root of minus one, were at first regarded as mere mathematical whimsy. Now they play a crucial role in the theories of fluid dynamics and electrical circuits. Group theory, used by Galois in the early nineteenth century as a means of studying mathematical symmetries associated with the solutions of polynomial equations, has subsequently found significant applications in the study of subatomic particles, in crystallography, in information theory, in photochemistry, and in the elucidation of certain complicated marriage systems studied by anthropologists. Non-Euclidean geometry, one of the great triumphs of abstract logical thinking, was a forerunner of Einstein's celebrated physical theories, which imply that the universe we live in is "curved" in the sense that parallel lines do not remain equidistant when extended into space. Graph theory, the mathematical study of abstract networks, was considered a rather esoteric kind of pure mathematics until recently when it was applied to problems in transportation, communications, urban planning, electrical networks and sociology. Graph theory is now studied either as pure mathematics or as applied mathematics, depending on one's point of view.

The Mathematical Sciences

The explosion of mathematical knowledge which has taken place in the twentieth century, coupled with the dramatic expansion in the range of possible applications, has splintered the mathematical community into a large number of more or less independent disciplines. Collectively referred to as the Mathematical Sciences, these disciplines are categorized under the main headings of Pure Mathematics, Applied Mathematics, Statistics, Operations Research, Actuarial Science, and Computer Science.[4]

Each of these major categories is further subdivided into specialties and subspecialties. For example a statistician who is helping to plan scientific experiments may have little in common with other statisticians who are designing surveys, making economic forecasts, or building stochastic models. Similarly, an applied mathematician who is studying shock waves in fluids may know very little about other branches of mathematical physics, and nothing at all about mathematical economics or mathematical biology. The mathematical aspects of computer science range all the way from basic numerical analysis to sophisticated theories of computer languages and artifical intelligence. Operations Research also takes in a large territory, ranging from standard optimization procedures to mathematical game theory and cybernetic control. Even Actuarial Science, which has been concerned with the mathematics of investment and insurance for hundreds of years, and which has so far resisted the modern trend toward ever-increasing specialization, is beginning to branch out in a number of new directions. And the grandmother of them all, Pure Mathematics, has succeeded in producing so many offspring that no one can possibly keep track of them.

> *"Our intellectual wealth is becoming truly embarassing. The mathematical universe is already so large and diversified that it is hardly possible for a single mind to grasp it, or, to put it another way, so much energy would be needed for grasping it that there would be none left for creative research. A mathematical congress of today reminds one of the Tower of Babel, for few men can profitably follow the discussions of sections other than their own, and even there they are sometimes made to feel like strangers."*

Those words were perfectly true when George Sarton wrote them in 1936, and they are even more appropriate to today's situation.

Despite this fragmentation into sub-specialties, which has been so characteristic of modern science, a rather remarkable process of integration and unification has been taking place at the core of mathematics during the last few decades. This development is illustrated most clearly by the work of Nicholas Bourbaki. Bourbaki served as a general for the French in the Franco-Prussian war and surrendered 10,000 troups to the combined militia of Geneva and Lausanne! The word "bourbaki" used to occur in a joke which the sophomores of the Ecole Normale Superieure played on the freshmen. It was assumed as a collective name by a group of young French mathematicians in the mid-thirties who bound themselves into a disciplined secret society which set out to write a definitive treatise on the Elements of Mathematics. In so doing they isolated a few mathematical concepts, such as algebraic or topological structure, and used them to effect an extraordinary unification of the core of mathematics which brought previously diverse parts of mathematics into much closer relationship and gave them new and deeper significance. Although none of the founding fathers still belong to the group, Bourbaki lives on and has been actively writing, constantly revising "his" treatise. Presumably Bourbaki will continue to produce as long as France remains a major mathematical power. Nicholas Bourbaki is one of the most fascinating and significant phenomena in the whole history of science.

The Crisis

> *"It was true during ancient times, it was true during the Renaissance, it was true during the lifetime of Newton, Leibniz, Euler, Gauss, Lagrange, Laplace, and Cauchy, it was true during the era studded with the names of Hamilton, Riemann, Cantor, Hilbert, Lebesgue and Poincare, and it is still true today that mathematics represents the supreme achievement of the human intellect, the triumph of logic over science by mathematically-based methodologies. And yet, there are many observers, some within mathematics and some - like myself - on its fringes, who feel that mathematics is in a state of deep crisis which calls into question her ability to survive for long as the queen of sciences. This may sound unduly alarmist and pessimistic. Perhaps it is. Be that as it may, I believe that mathematics will not be able to weather the stormy waters which she has entered without a profound change in her aims and orientation.* - A. Zadeh[5]

Our description of the recent vigorous development of the mathematical sciences and their applications might lead one to conclude that mathematics is in a very healthy state. In over two thousand years of its history, it has never been in a more creative or more productive phase. Nonetheless, there is a widespread feeling within and without the mathematical comunity that all is not well. The very diversity of modern mathematics has created extraordinary problems of communication and coordination. Signs and portents of this which became explicit during the Study can be summarized as follows. Most of them appear in other Western countries as well:

-though our civilization is largely based on mathematics, most Canadians profess to have no real understanding of mathematics and consequently feel uneasy when faced with a mathematical argument.

-there is widespread dissatisfaction with the teaching of mathematics in the schools; the "New Mathematics" confuses many parents and teachers; there is a measurable decline in computational skills; many university professors assert that first-year students in engineering and science are inadequately prepared in mathematics.

-the graduates of university mathematics programs are criticized by managers in industry and government for their inability to communicate with non-mathematicians, to work cooperatively with others, or to tackle unfamiliar problems with zest and courage.

-in the last three years many PhD's in mathematics have failed to find employment which they considered suitable; this has caused consternation among their professors and has cut the nerve of many programs of graduate study which until recently were in exuberant expansion.

-the widespread discussion about science policy, typified by the Senate Committee on Science Policy, has raised questions about the allocation of funds between research and development; these questions cause anxiety among the many academics whose main raison d'etre has been to do research in abstract mathematics.

-the users of mathematics in the other sciences, in engineering, in business and government, whose need of mathematics has been steadily escalating complain that the mathematical community is closed and unresponsive to their pleas for help.

At first sight, it seems very difficult to encompass all of these considerations within a single context. With so many different people working in so many different areas of research and application, it is not surprising that the mathematical community seems somewhat "disintegrated" at times! Most of its members, scattered about in a wide variety of working environments, see only a tiny portion of the universe of mathematical activity, and they plan and act accordingly. Each group has its own problems, and its own objectives. Members of different groups seldom come into direct professional contact with one another. What is it, then, that binds them together? And where is that community of interest that allows us to speak of a "mathematical community"?

The Mathematical Ecosystem

Perhaps an analogy would be useful at this point. Think of an ecosystem, involving many different species of plants and animals interacting among themselves and with their physical environment. An ecosystem is a vibrant, organic, interactive process. The different species are interdependent in many different ways, but primarily by means of food chains and food webs which can be extraordinarily subtle and complicated. If anything is allowed to destroy these food webs, the entire ecosystem suffers.

Similarly, in the modern world, there are many different kinds of people who are professionally concerned with mathematics - teachers, researchers, and users of mathematics of many different types. These are the different species within our "Mathematical Ecosystem". They are linked together not by nutrition, but by ideas and information. Instead of a food web, we have a communications network. Moreover, the health of the entire Mathematical ecosystem depends critically on the proper functioning of this communications network. As one of our correspondents wrote:

> "I believe that a central problem in science everywhere, and particularly of mathematics in Canada, is one of communication. This is a question not only of facilities for publication, but also of the means by which mathematical ideas are spread and retrieved at all levels, from the most advanced research to teaching in the elementary schools. The efficiency of this information system is one of the most important factors that will determine the future of Canadian mathematics."

As we shall see, the communications network in mathematics involves more than just information. It involves the transmission of attitudes and values as well.

The remainder of this chapter is devoted to a brief description of the five main components of the Mathematical Ecosystem, which we identify as follows:

(i) Schools - Elementary and Secondary
(ii) Community Colleges
(iii) University Mathematics Departments
(iv) University Science Departments
(v) Environment - Business, Government, Industry.

Particular attention will be paid to the linkages between these five components in terms of the communications network mentioned above.

(i) Schools

It is in elementary school, that children form their first impressions of mathematics as a separate discipline - impressions which are often so deeply etched in their minds as to stay with them for the rest of their lives. Mathematics is seen by a few as a handy tool as well as a source of pleasure and satisfaction, but too frequently, children are intimidated, confused, or frightened by their experience with mathematics in school. Negative attitudes acquired at this early stage will constitute an enormous obstacle to the effective and sensible use of mathematics throughout their lives.

Only too often the teachers themselves had difficulty with mathematics when they were young and developed an aversion for the subject - particularly if it was taught to them as an endless catalogue of dry, boring topics, devoid of any human interest. Such feelings in a teacher are, almost inevitably, transmitted to the children. So these attitudes are propagated from generation to generation. We suffer from the effects of a positive feed-back loop!

Elementary school teachers, who have always had to teach basic arithmetic whether they liked it or not, are nowadays supposed to help their pupils achieve an understanding of many things which they themselves never really understood. They are asked to teach arithmetic in different number systems, to use the language and concepts of set theory, and to introduce the rudiments of algebraic manipulation and geometric construction - while ensuring that their pupils still learn all the ordinary arithmetic skills! A tall order indeed, especially since there is a great deal of confusion among teachers as to the basic objectives of the new mathematics program which were introduced into the schools during the sixties. They would like to know why this particular kind of "mathematical understanding" is important, and to whom. They could teach the subject better if they knew what they were supposed to accomplish. As it is, how are they to judge whether or not they are doing a good job unless they achieve some meaningful communication with the other components in the Mathematical Ecosystem?

In secondary schools, mathematics takes on a more formal flavour, as adolescents learn to wrestle with quadratic equations, congruent triangles, logarithms, coordinates, and trigonometry. For some students, these mathematical exercises are a form of mental yoga, and they thrive on it. Many others, however, find their high school encounter with mathematics a deeply frustrating experience. It never quite succeeds in engaging their imaginations. They can neither enjoy it as a thing in itself, nor can they relate it to anything else which they do enjoy. They believe that algebra and geometry are important [6], but only because they are constantly being told by their elders. They do not feel it in their bones. The only explanation they are likely to get is, "Wait and see; someday you will understand why these things are important". But they have no desire to wait and see. Many just want to drop mathematics as soon as it seems safe to do so.

In the past, when only a minute fraction of high school graduates ever actually needed to make use of their mathematics (by becoming engineers, physicists, or mathematicians, for example), this state of affairs may have been tolerable. But times have changed. It is no longer possible to guess in advance which high school students might need to have a good grasp of fundamental mathematical principles, in order to pursue careers in transportation, communications, manufacturing, agriculture, resource management, environmental studies, marine biology, cancer research, and economics. The far-reaching and revolutionary implications of this fact are only beginning to be felt at the high school level.

Perhaps it is this appreciation by students or it is the pressure from their elders which explains an interesting observation in Ontario. Until a few years ago most high school students in Ontario were forced to study mathematics from Grade 9 through Grade 12. According to the regulations of the Department of Education, mathematics has been completely optional for the past five years. Many persons expected that there would be a sharp decrease in the number of students enrolling in mathematics. Such has not been the case. Indeed, if we count Computing Science as mathematics, in many schools the number of students taking mathematics has actually increased.

Together with the growing understanding [7] of the importance of mathematics in society there is a feeling that universal mathematical literacy should be one of the principal objectives of our whole school system - not just in terms of imparting basic mathematical skills, but also in terms of inculcating in every person an understanding of how mathematical methods are currently being used (and abused) in science and in society, and what part they could play in the future.

Too often mathematics in high school is treated as a "spectator sport" rather than as a creative intellectual activity. Too often it is presented as a self-contained subject, self-motivated and divorced from other fields of human endeavour. Too often the students' satisfaction is derived, not from the joy of intellectual insight, but from getting "marks". Teachers and students alike are moved to question the relevance and the purpose of much of what is taught. The following remarks taken from a letter addressed to the Mathematics Study by the Head of Mathematics at a small Ontario Collegiate, illustrate this concern:

> *"My feeling is that the present courses have been set up for able students headed towards pure mathematics. Most students are not headed in that direction. The courses should bear the majority in mind and core material should be simple enough to encourage them to bloom. In addition, we believe that many bright students are bored due to the lack of practical type problems in the texts. As they progress through elementary and secondary school we feel that a very real effort should be made to include several problems that apply to the topics under discussion and that the students could feel came from their 'real world'."*

The desirable goal of universal mathematical literacy will only be achieved when teachers at the high school level are in communication with those who use mathematics in today's world - the mathematical practitioners - so that they can begin to breathe an air of practicality into their classroom teaching and regain the interest of some of their more skeptical students.

(ii) Community Colleges

The Community Colleges, in one form or another, emerged throughout Canada during the mid-sixties. Designed in part to provide a post-secondary alternative to the university, these institutions vary widely in concept from province to province.

In Quebec no one is allowed to enter a university without graduating from one of the two-year CEGEPS (College d'Enseignement General et Professionnel), while in Ontario the concept of the CAAT's (College of Applied Arts and Technology) explicitly rejected the task of preparing students to go on to University, British Columbia has a system which combines features of both Quebec and Ontario in that students may enter university directly, or they may begin at a Community College and then transfer to university, or they may enroll in various other college programs as an alternative to university.

It did not take long for Community Colleges to notice their students were having very high failure rates in mathematics. It was not just the formulae that they had forgotten; they had forgotten even the basic ideas on which the formulae were based. Perhaps they had never understood these basic ideas in the first place. They could not add and subtract positive and negative integers. They were unable to work with negative or fractional exponents. They did not know how to get a common denominator in order to add two fractions.

Because graduates from their technical programs try to find work as technicians in industry or in government, Community Colleges are usually careful to hire teachers who have some practical experience. These men and women are just as aghast as their more theoretical colleagues at the lack of mathematical competence of so many of the incoming students, which goes hand in hand with their lack of mathematical confidence. Many colleges are making strenuous efforts to establish contacts with the secondary schools in their own areas, to bring the message home to the high school teachers. High school administrators, still sensitive to similar criticisms levelled at them by parents, are slow to admit that such a problem exists. But the facts speak for themselves and the truth is beginning to be acknowledged. Given the opportunity, many of the colleges would be quite eager to cooperate with the high schools in trying to tackle this problem. But first there has to be a reliable channel of communication established between them - another link in the Mathematical Ecosystem.

(iii) University Mathematics Departments

Every university in Canada, indeed in most countries, has a Department of Mathematics. There are, in addition, a large number of Computer Science Departments in Canadian Universities, together with a handful of Applied Mathematics Departments, and a very few Departments which are devoted to Statistics, Actuarial Science, or Operations Research. The University of Waterloo has gone so far as to set up an entire Faculty of Mathematics with five constituent Departments: Pure Mathematics, Applied Mathematics, Combinatorics and Optimization, Applied Analysis and Computer Science, and Statistics. The Mathematical Sciences are typically housed in a single Department (the Mathematics Department), with two exceptions: Computer Science and Operations Research, which are, for one reason or another, largely ignored by Canadian Mathematics Departments. Computer Science usually constitutes a Department by itself. Operations Research is generally found in the Faculty of Commerce or the Faculty of Engineering.

Normally, any of these departments will offer (a) a program for students specializing in the areas of particular competence of the Staff, and (b) a set of "service" courses for students whose primary interests are not in a mathematical science but who need some specific mathematical topics as working tools or as background knowledge for the pursuit of their principal interests.

Many observers of the mathematical scene in English-speaking Canada would agree that in the period 1930 to 1960 the University of Toronto was the dominant Department providing a paradigm by which other Canadian mathematics departments judged themselves and were judged by others. Consequently, the function described above under (a) absorbed so much time, energy and interest of the staff that in most universities little systematic attention was paid to function (b). The main sequence of undergraduate course offerings was designed to prepare students for graduate study, research and university teaching. In recent years the demand for mathematics teachers at all levels has been slackening off and many graduates find that their university training did not prepare them well for non-academic employment. Many recent PhD's in mathematics are unable to locate permanent jobs which would make good use of their eight or ten years of intensive study since high school graduation. In Mathematics Departments across the country, enrolments in honours programs and graduate programs are holding steady or even dropping as the job market tightens. Mathematics professors feel the need to establish better channels of communication with potential employers of mathematicians.

But if the honours programs and the graduate programs are taking a bit of a beating, the service courses are booming as never before. Students of Physics, Chemistry, and Engineering are no longer the only ones who have to study mathematics as an integral part of their program. In almost all of the social sciences - psychology, economics, political science, sociology, geography - at least one course in basic statistics is essential, and courses in calculus, linear algebra, and computer science are strongly recommended.

Students of biology and medicine likewise feel the need to know something about college-level mathematics.

Although people in other disciplines may have to learn mathematics in order to keep abreast of their fields, in the past students of mathematics were not forced to learn anything else but mathematics. This is one reason why many user departments are now teaching their own mathematical service courses to their own students. They feel that professors of mathematics just do not know enough to be able to communicate effectively with their students. Unfortunately, they themselves often know so little mathematics that they end up presenting it to their students in a sketchy cookbook course which does not teach mathematical thinking. Wherever this type of situation occurs, students are short-changed. Clearly there needs to be more communication in both directions between members of the departments in the mathematical sciences and the professors in the "user" departments.

It is now common-place to note that the advent of the electronic computer has greatly enhanced the possibility of making effective use of a wide variety of mathematical techniques. In the late 1950's and early 1960's many universities in North America created Departments of Computer Science. Despite identity in name, these Departments differ considerably among themselves. Some concentrate almost exclusively on data processing, others are more concerned with computer technology (both hardware and software), and still others have a significant theoretical-mathematical component. Hints about the present orientation of a particular Computer Science Department can sometimes be gleaned from its genesis: did it develop out of a business school? an engineering faculty? a mathematics department? or did it come from two or three different directions at once?

Whatever its origins, the Computer Science Department usually has little to do with the Mathematics Department, where standard courses in calculus, linear algebra, statistics and differential equations are often taught as if the computer did not exist. Indeed, many mathematicians seem to regard the computer as a glorified adding machine which carries no sigificant implications for the future development of mathematics. Numerical computation is deliberately avoided by such mathematicians as being less exact and/or less aesthetic than the traditional methods of algebra and analysis. It is not surprising that computer science has developed as an autonomous discipline with its own characteristic approach to problem-solving, based on the concept of a recursive algorithm - a stepwise procedure which produces approximate numerical answers by performing a very large number of arithmetical operations in a repetitive pattern.

The neglect of computation as an integral part of mathematics is already having unfortunate repercussions: (a) Students in service courses who are destined to use mathematics mainly (or only) in conjunction with a computer may have to relearn much of what they are being taught about higher mathematics in order to see whether it can be adapted to computer usage. (b) Because of their ignorance of computer methodology, students who are concentrating in mathematics will have added difficulties in obtaining non-academic jobs where they can make use of their mathematical knowledge. (c) Students who are more inclined towards mathematical research are not encouraged to speculate about a host of fascinating and important theoretical questions, ranging from the convergence and stability of numerical methods to the theory of recursive functions and the nature of artificial intelligence. (d) Meanwhile, existing difficulties in communications between mathematicians and users of mathematics are bound to get worse if mathematicians continue to ignore the computer, which has become the users' main tool in bringing mathematical thinking to bear on problems arising in other fields.

A very large proportion of computing time is devoted to statistical calculations. Indeed, statistics is probably as widely used and as important for applications as computing. Nonetheless, there are only a few[8] Departments of Statistics in Canada. Most universities have a group of theoretical statisticians within the mathematics department and a number of applied statisticians scattered among many other departments. According to information coming to the Study from many sources, the condition of statistics in Canadian universities is regarded as "unsatisfactory" nearly everywhere. On the one hand, rigorously trained statisticians are very critical of the statistical methods and procedures employed by many students and researchers in the biomedical and social sciences. It is alleged that, at best, they are frequently wasteful and inefficient and, at worst, grossly misleading and erroneous. On the other hand, students in psychology and biology assert that statistical theory as it is taught by professors in mathematics departments is so austere as to be frightening and so abstract and lacking in practical illustrations as to appear quite unrelated to any of their interests. These problems are universally recognized and, happily, several creative innovations[9] are now underway.

University departments in the Mathematical Sciences occupy a strategic focal point within the mathematical ecosystem. They bear almost the full responsibility for research to push back the limits of mathematical knowledge. However, at the same time they must relate effectively to the schools to ensure universal mathematical enlightenment. They must also maintain efficient channels of communication with other scientific disciplines and with industry, business and government in order to be aware of societal needs and to prepare their graduates for successful non-academic employment.

(iv) University Science Departments

Much of the new mathematics of the eighteenth and nineteenth centuries was invented in response to the needs of physics. Without doubt, physics is the most mathematically-intensive subject apart from the mathematical sciences as such. Then would follow theoretical chemistry and parts of engineering - particularly electrical and mechanical engineering. In these areas there are some mathematical models - such as Newton's for the solar system - which are so accurate that calculations based on them agree with experiments almost as closely as can be measured.

Except for those who end up in an R and D (research and development) environment, most engineers have managed to get by in the past with an arsenal of mathematical formulae which can be applied almost unthinkingly to problems which arise in everyday engineering practice. Those formulae, like simple recipes, can be used without any understanding of how they came about or why they work. The reason why this "cookbook" approach often succeeds in engineering is because the mathematical equations representing physical and chemical processes are such remarkably exact representations. For all practical purposes, the mathematical models behave exactly the same way as the real world. Nevertheless much engineering work increasingly requires a full understanding of the mathematical basis of the formulae involved. Moreover, with the advent of the computer, many engineering problems are now being tackled which defy the cut-and-dried formula approach. Among professional engineers there is frequently considerable tension between those who believe that the simple old-fashioned rule-of-thumb methods are sufficient and those who insist that mathematical ideas of ever-increasing sophistication have been and will be essential to enable the engineering profession to remain in the forefront of Canadian technological development.

When we pass from sciences such as physics and engineering to those such as biology and economics, we find a completely different state of affairs. In these non-mechanistic fields, the precise behaviour of the phenomena to be studied is simply not known. As a result, the application of mathematics in these sciences requires much more in the way of mathematical thinking. Underlying assumptions have to be constantly borne in mind. At any time, the formulae that are being used may prove to be inadequate for one reason or another; if so, they will have to be modified or replaced. There is always a clear distinction between the mathematical model and the real life situation that is being modelled, because the representation is so crude; the model is an extremely simplified representation of reality, which functions more as a valuable analogy to be explored than as a detailed replica to be believed. Unfortunately, the spirit of mathematical thinking as expressed in mathematical model-building is noticeably absent from most service courses in mathematics. Those who teach service courses frequently seem to think that there are only two ways to do it: either to give the full theoretical treatment, or to teach a bagful of handy little tricks. In either case, creative mathematical thinking is almost totally absent. The student never learns to "mathematize".

Many of those who want to apply mathematics in the non-physical sciences are victims of their own miseducation. Some of them have never learned how to think mathematically. Through school and university, mathematics has acquired such a mystique of being unarguable and unassailable that it is more often used as a blunt instrument than as a delicate tool of investigation. Yet there are a great many who would use mathematics more intelligently, if only they could find a mathematician who is willing to help out at the most difficult level - not as a technical specialist, but as a co-thinker.

(v) The Environment

The role of mathematics in business, industry and government will be the topic of the next chapter. Occasionally, good operations research, statistical or computing teams are saddled with a management that does not understand how they differ from plumbers. Only the actuaries are held in high esteem since they have achieved the status of an admired profession on which the financial success of the life insurance business has depended. Increasingly, however, mathematics is recognized as indispensable to business and government. Managers who were initially disenchanted by the failure of mathematics to supply instant answers to all their problems are becoming a little more patient, since they are still faced with the same problems, which are unavoidably mathematical in nature. Computers have been employed for more than a decade, soaking up information and structuring it into a more usuable form. As a result, problems which were previously intractable because of lack of data and lack of definition are now showing more promise. A mathematician at Bell Canada told us, "The challenge of the 60's was to get the data. The challenge of the 70's is to use it."

The low level of mathematical literacy in Canada is one of the biggest obstacles preventing the more effective use of mathematics in developing technology, planning for the future, minimizing environmental and social impact, rationalizing transportation and communications, and improving public services. Most people who are not mathematicians will go to any length to avoid looking at a mathematical symbol or listening to a mathematical line of reasoning. The opposite extreme - swallowing some half-baked mathematical theory in a rush of blind faith - is occasionally seen in business or government, where it usually backfires rather badly, following the old adage: "once bitten, twice shy". A worship of things mathematical is not an uncommon manifestation of mathematical illiteracy and can sometimes be used by decision-makers to dress up their decisions with mathematical claptrap in order to make them look more scientific and less open to criticism. Such abuse will do nothing in the long run to enhance the appreciation of mathematics as a socially useful tool.

The chief obstacle to the more extensive use of mathematics in Canadian business and industry is the lack of well-qualified people to work in these areas of application. Good applied statisticians, operations researchers and mathematical model-builders are hard to find. At present, most workers in these fields are engineers or scientists who picked up their knowledge of mathematics through university service courses, and who later migrated into their present occupations because they found them challenging, exciting and useful. University majors in mathematics, on the other hand, seldom hear about the kind of work that is involved, so they either bypass it altogether or else they enter it without being prepared for the shock of plunging into a heavy political atmosphere where problems are clamouring for a quick solution by any method, no matter how inelegant, and where communications with fellow workers and with superiors is extremely important and extremely difficult. With a few notable exceptions, contact between academic mathematicians and non-academic mathematicians is almost non-existent; as a result, university professors generally know very little about the conditions of work in non-university positions. How are the students to learn if the teachers do not know? Most Canadian universities do not even offer refresher courses to upgrade the mathematical skills of industrial employees, or special introductory courses to acquaint managers with the usefulness of looking at certain practical situations in a mathematical way. Most companies and government offices gave an emphatic "yes" when asked whether such courses would be well-received, provided that they were practical, to the point, and well taught.

Similar observations apply to the use of mathematics in the municipal, provincial, and federal levels of government. Engaging in urban planning, providing public health protection, managing the economy, protecting the environment, regulating the use of resources, satisfying transportation needs, maintaining a nation-wide postal service, and doing basic research in agriculture, aeronautics, nuclear energy, and marine biology - all of these endeavours require a good deal of mathematical know-how, quite apart from the routine tasks of data collection and analysis. You are more likely to find mathematicians in Government than in industry, usually acting in a consulting role, though they are still greeted with suspicion and misunderstanding in many cases. Decision-makers who do not understand mathematics will sometimes reject perfectly good solutions if they feel uncomfortable with the mathematical terminology and symbolism. One senior operations research worker in the federal government quoted a saying of Stafford Beer: "A manager would rather live with a problem he can't solve than with a solution he can't understand." That puts the point in a nutshell.

Conclusions and Summary

In most cases fundamental attitudes towards mathematics are firmly established by the end of high school. Some high school graduates are capable and confident in mathematics, but many others feel tremendously insecure and even antagonistic towards the subject. They resent having to work so hard at a subject which gives them nothing in return but a feeling of humiliation every time they get something wrong, which happens more and more often as they become more and more befuddled. Those who become parents and teachers will carry their attitudes with them into the home and the classroom, where they will pass them on to the next generation. Those who go into business or government work will approach the use of statistics, operations research, and model-building with the same attitude that colours their own strong feeling about mathematics. Those who go on to become technicians or scientists will often find mathematics a major stumbling block, as unfamiliar mathematical concepts and techniques continually creep into their work. In many cases their schooling in mathematics will turn out to be worse than inadequate; skills and techniques which were never understood and seldom used are quickly forgotten, but negative attitudes persist and become a serious obstacle to further learning. On the other hand, those who go on to obtain a university degree in mathematics frequently discover that they are unable to communicate effectively with non-mathematicians, whether in the classroom or on the job. Some of the younger mathematics graduates deeply regret the fact that mathematics, which should be one of the most onward-looking disciplines, has instead become one of the most inward-looking. They feel cutoff and isolated from the world around them, and although they enjoy mathematics immensely, they often wonder whether a better balance between theory and practice could not be found. The results of the Questionnaire show that many graduates in mathematics feel that their university failed them in this regard.

There are so many centres of mathematical activity, and there is so little communication between people in different centres! The major trouble spots in the Mathematical Ecosystem seem to occur on the interfaces between the various components of the system. The communications network is in a precarious state. Policies are planned and executed by men and women who only see a small portion of the total system, in consultation with others whose vision is also limited in scope. Little or no feedback is obtained from those who move through the system, so the ultimate effects of such policies go unnoticed by the people who framed them in the first place. The result is a hodgepodge of uncoordinated efforts acting at cross purposes. It is remarkable how many pressing issues related to the role of mathematics in Canadian society are coming to a head at approximately the same time. It no longer seems possible for any component of the Mathematical Ecosystem to function effectively in isolation. Awareness and communication seem to be the key issues.

One cannot reasonably expect that persons who have no knowledge or understanding of mathematics will be able to delineate the complex patterns and interconnections of the mathematical ecosystem. One could expect that the professors in university departments of the mathematical sciences, whose very livelihood depends immediately upon the health of that system, would devote much time and effort to so doing. In fact, however, most able mathematicians are seeking personal enjoyment, and fame among their peers, by single-minded pursuit of esoteric abstract knowledge; so they make little effort to understand the total complex social and historical role of mathematics and still less do they attempt to explain these factors to non-mathematicians.

In the course of the Study no message was clearer than the following: the mathematical community must become more out-going, devoting more energy to communicating meaningfully to users of mathematics on their terms; the users of mathematics must break down their mental blocks and undertake the hard intellectual effort necessary to use mathematics as a truly effective instrument.

Notes: Chapter II

1. Professor Beaumont is Associate Dean of the Faculty of Mathematics at the University of Waterloo. The quotation is from a speech he gave at the University of Colorado in 1973. It is to Professor Beaumont that we owe the idea of the mathematics-industry seminars. He was responsible for organizing five and chairing four of them.

2. These disciplines are discussed in a brochure setting forth the philosophy of the Industrial Management and Applied Mathematics Group of the Catholic University of Louvain, dated February 1975, and entitled, "Les mathematiques appliquees: Une carriere pour l'ingenieur?"

3. A.N. Whitehead, Science and the Modern World, p. 41, Cambridge University Press, Cheap Edition, 1933. First published 1926.

4. It should be remarked that many of those who are actively working in statistics, actuarial science, operations research, or computer science, would not describe themselves as mathematicians or their work as mathematical. Such people regard themselves as scientists, technicians, or businessmen, for whom mathematics, is, at best, only a tool.

   In many cases, mathematics is not always used as a tool, except at a very rudimentary level. A good number of statisticians are working on nothing but descriptive statistics, which seldom requires any mathematical analysis at all. Some operations researchers are management consultants who use no mathematical apparatus whatsoever. Many actuaries are really full-time administrators, and most computer scientists are dedicated either to computer technology or to data processing, both of which involve mathematics only in a peripheral way.

Nevertheless, it is still true that there are large numbers of people in each of these fields who use mathematical thinking as an essential part of their work. And, without this mathematical core, none of these fields would exist as they do today. That is why they are called "Mathematical Sciences" - not to belittle the non-mathematical aspects, which are quite prominent, but to emphasize the vital links which they all have with mathematics.

5. A. Zadeh is an American engineer who was quoted in the C.M.C. Notes, November 1972. pp. 2-5.

6. A.J. Skalfuris, Head of the Mathematics Research Centre of the Naval Research Laboratories in Washington, D.C., in a letter to Physics Today (March, 1975, p. 15) complains of the scarcity of physicists with a wide-ranging command of mathematics. Too often, he states, the inadequacy of the mathematical techniques at one's disposal dictates the problem one attempt to solve.

   G.B. Kolata in an article in Science, Vol. 187, p. 732, (1975) entitled "Communicating Mathematics: Is it Possible?" stated that two-thirds of the mathematicians approached by Lynn Steen at the International Congress for Mathematicians at Vancouver in August 1974 were uninterested in explaining mathematics to those outside the field.

7. For example, according to a report emanating from Edmonton and described on April 12, 1975 in the Globe and Mail of Toronto, a survey in which 1,500 high school students were asked to rate 25 subjects in order of importance, found that they rated mathematics first and English (the local language) second. (To satisfy the curiosity of the reader, we report that, in order, the next were social studies, sex education, consumer education, biology, contemporary problems and Canadian history.)

8. To our knowledge, only at the Universities of Manitoba and Waterloo.

9. The American Statistical Association has announced an ambitious program to develop a modular approach to teaching introductory statistics. A group at Queen's University has proposed a similar project. The Statistical Science Association of Canada has faced up to these questions very directly at its annual conference.

Chapter III

Mathematics In Business, Industry and Government

> *"Some people see mathematics as a mysterious supertool "to save this world from the bungling bureaucrats". To others, mathematical techniques seem to be just "the intellectual trappings of boy-wonders who have never met a pay-roll, never run for Sheriff, and shouldn't try to run a country". The truth, no doubt, lies somewhere in between. If you experts, you applied mathematicians, you systems theorists, can find some truths that fit the facts of our age and the dilemmas of our nation, and tell them so they can in fact be understood, then you may not have the applause of your mathematical colleagues, but you can help to shape the future of your community. I am convinced of that. However the analyst must be content to see this influence through a slow spread of new ideas, rather than through an immediate impact on next week's decisions. He must therefore recognize his analysis as useful even when his conclusions are rejected. If he cannot get this across to his students and colleagues, I fear they will be frustrated in government service."*
>
> \- The Honourable C.M. Drury[1]

The present chapter is largely based on input to the Study from the eight seminars described in Chapter I, supplemented by ideas from briefs and interviews. The main themes are:

-the extraordinary increase in the applications of mathematics in business, industry and government in recent decades;

-the diversity of mathematical techniques actually utilized appears to be limited chiefly by the absence of persons with wide-ranging mathematical competence to assist in the formulation and analysis of the problem at hand;

-the training of mathematics majors does not develop in them the ability to communicate readily and sympathetically with others or to venture upon the mathematical modelling of the vaguely defined and messy problems that are thrown up by reality;

-the contention that managers are distrustful of mathematically trained persons, do not sufficiently encourage their participation in analyzing problems, and are often unwilling to make any real effort to communicate with them;

-in all the Seminars there was a very positive feeling of cooperation, and unanimity that the sort of interaction which they allowed should be energetically pursued and widely emulated;

-practical steps, which may involve some financial outlays, should be taken to encourage the interchange, for short periods, of persons between universities, community colleges, business, industry, and government to enhance mutual understanding of what types of mathematics are now available, what are practical problems to which they could be applied and what attitudes and skills are needed to overcome the various communication gaps.

However, before launching into an account of how Canadians viewed these issues, we shall report on a discussion involving mathematicians from several countries which took place at the International Congress for Mathematicians in Vancouver in August, 1974.

A World Perspective

In 1970 a group of mathematics students in the University of Bielefeld in north-west Germany became concerned that their training was not preparing them adequately for the types of jobs open to them. It was during a period of great student unrest in Europe, so their concern was taken very seriously and a series of Seminars was organized between October 1970 and January 1971, to each of which was invited a "mathematician" working in industry who briefly described the nature of his work and the adequacy or inadequacy of his university training as a preparation for his Berufspraxis - the practical activities of his profession. The speaker was acutely questioned with the well-known German gruendlichkeit (thoroughness!) There were speakers for IBM, ESSO, Telefunken, an Insurance Company, the Farbenfabriken Bayer AG and the Battelle Institute in Frankfurt. The talks and a record of the discussion were published under the title Materialen zur Analyse der Berufspraxis des Mathematiker for the Projektgruppe on "Mathematik in der Industriegesellschaft" by the Mathematics Faculty of the University of Bielefeld. The recurrent themes of discussion were essentially the same as those which appeared in the five Seminars organized for business and industry by the Mathematics Study.

From that original student initiative, there emerged a University of Bielefeld Commission on the Mathematisation of the Individual Sciences. ("Mathematisierung der Einzelwissenschaften") which was formally constituted on November 16, 1973. The membership of the Commission includes biologists, chemists, historians, mathematicians, education professors, philosophers, psychologists, physicists, lawyers, sociologists, linguists, and theologians. The Commission laid out a comprehensive work-plan involving monthly colloquia on mathematics and individual disciplines such as sociology, chemistry, psychology, teaching, physics, law and biology. They also set in train the production of a very detailed Questionnaire concerning the practical experience of mathematicians outside of the University. A draft version of this Questionnaire was available at the ICM in Vancouver. It probes many of the same issues as are raised by the Mathematics Study Questionnaire.

At Vancouver Drs. Bernehlm Boos and Ulrich Knauer, who represented the Bielefeld Commission, organized a very successful round-table discussion. Here are a few excerpts from that discussion.

A Canadian We mathematicians conduct our work by standards that are false. We talk about the wrong things. Here are examples of distortions in our evaluation of our own work: (i) We deal too largely in proofs and solutions of problems and too little in discovery of phenomena. Discovery of phenomena which used to be exciting for 19th century mathematicians is hardly of concern in the 20th. One exception is morphogenesis or catastrophe theory [2] which has got a lot of us really turned on. (ii) We overemphasize difficulty. Some of the most important discoveries in mathematics are simple. In the 20th century, difficult problems are given great importance just because they are difficult. However, a user wants a practicable answer. He will not admire it just because it is difficult.

A Frenchman With us nearly all university mathematicians work in pure mathematics and there is very little applied mathematics taught. This is a real problem since we are unable to teach students mathematics in the way they need to work in production. Since 1968, there has been something like a fight between the universities and the government. The principal way the government has for acting is to give more money to support the direction in which it wants things to move. However since 1968, the individual universities have some autonomy and are not always responsive to government wishes.

A Cuban Before the revolution, we had almost no mathematics at all and started to build a school of mathematics in Havanna in 1962 - just twelve years ago, when nearly everyone who really knew anything about mathematics had left Cuba. They were bourgeois. Our main problem is to train young people. Now we have mathematics departments in three universities. At first, we had no applied mathematics at all, but like the socialist countries, we are very much interested in applying mathematics to social and organizational problems. However, in a developing country, to put all the stress on applied mathematics is a bit dangerous because you can lose your chance of future development. Nonetheless, it is clear to us that in the present epoch we must do much more applied than pure mathematics. Before the revolution, the Americans did everything for us. So for example, all Cuban engineers were only maintenance and not design engineers. Now we are just starting to apply such topics as statistics and operations research to industry.

A Russian Many of our professors at polytechnics and universities want to teach only in pure directions without applications. Many students - future biologists and engineers - do not understand why they need to study mathematics.

We have a second difficulty. Specialists in the natural and engineering sciences do not always make use of available mathematical knowledge. So last year, under the chairmanship of Kolmogorov, we organized an unofficial commission on mathematics education in the University. It will (i) study the actual situation in all the universities, (ii) develop an appropriate mathematical curriculum for pure mathematicians, for engineers, for biologists, etc.
(iii) initiate discussion between mathematicians and professors in the other disciplines.

At Ivanovo, which is the centre of the textile industry, there is a good seminar for the staff of the mathematics department about mathematical problems in the textile industry. This has led to new mathematical problems, some of which have been solved, and also to very good interdisciplinary work. Within two years, all the students have come to know that mathematics is very important for the textile industry and interest in pursuing mathematics has increased markedly among the students.

It should be compulsory that every future mathematician know something more than pure mathematics. At Moscow University every student must have some applied course, either in mathematical biology, mathematical economics, etc. My colleagues, Krylov and

Valutov, lecture on mathematical biology; Soloviev, on reliability theory. It is not only mathematics majors who attend these lectures. Among our students, the number who wish to study in an applied direction is increasing every year.

<u>An East German</u> I think this question which is being discussed everywhere now - the relation between pure and applied mathematics - is one of the most interesting for mathematicians who are concerned not only to prove theorems but regard mathematics as a science for the people. For traditional reasons, we have lots of pure mathematics in our country and very little applied mathematics. In the early sixties, we discussed this question from the political point-of-view since, as a country, we are interested in building up socialism. We asked a few young mathematicians to study in certain universities in the USSR where they could learn the disciplines which are important for applied mathematics. For example, Gnedenko had some of our able young mathematicians as students and they are now working in this direction. Now after twelve years, there is, in my opinion, a good mixture between pure and applied mathematics. However, it is not enough to teach applied mathematics in the university. It has to be combined effectively with actual practice in industry. The mathematicians who discussed this concluded that it is necessary to put some good mathematicians into industry to work on industrial problems. The first experiences with this indicate that we are right - the only method to get a real unity between mathematics and the problems of engineers is to work together.

Indeed, there can be surprising, almost unbelievable results. I personally am an algebraist and found that in designing a certain machine, I was led to use category theory. We had to model a situation involving probability, statistics and linear systems. We observed that the linear stochastic mappings formed an additive category. So we developed a theory of linear systems over additive categories - a purely algebraic question. It was a fairly long story, but at the end we had a model which could be treated by known methods. Being an algebraist, I looked at the problem from my point of view but stated the solution in language which the engineers could use. I did not have to explain category theory to them! I believe that a mathematician who is really interested in working on a particular application will try to apply his special discipline. It may give him a new point of view which works better than any that has been used before. I have concluded that the essential thing is not to teach students any particular parts of pure or applied mathematics, but in all our lectures to give them an impression of what mathematics can do in practice and the conviction that mathematics reaches out like this.

In his Life of Galileo, Brecht said that the only point of science is to help humanity to satisfy the needs of men. That is a bit too sharp in my opinion. Rather, the development of pure science and the development of industry have to go hand in hand. Hopefully, there will be two results: first, the level of mathematics in industry will rise; second, university professors who have worked for one or two years in industry will learn how to model reality and how to teach more effectively. We have great hopes there.

A Brazilian I quite agree that the crucial point is not teaching certain specific content. Rather, it is to enable the new generations of students to have an open mind towards applications. No matter what pure or applied mathematics they learn, if their attitude is open to the real world, they will be able to apply what they have learnt to relevant problems. I think this is a key point in guiding our policy for support to graduate students.

Mathematics Rampant

It was asserted in the previous chapter that the use of mathematics in many areas of Canadian society and,therefore, the need for mathematically trained persons has been increasing at an astonishing rate during recent decades. In the present section we assemble a few of the many illustrations of this fact which were brought to our attention in the course of the Mathematics Study.

From Tables 1A and 1B in Appendix I it may be seen that among approximately 1,500 respondents who majored in mathematics, 54% of the Bachelors, 40% of the Masters, and 13% of the PhD's are working in business, industry or government and approximately 90% of these are making use of their mathematical training in their profession - with computing, actuarial and business mathematics and statistics being the most commonly used types of mathematics.

When Bell Canada decides to change the rates for Long Distance Telephone calls, how does the company decide on the schedule? In the good-old-days it was simple enough. A rough estimate was made of the actual average cost of the service and management set the rate at some multitude of this, perhaps somewhere between 2 and 10, depending on their judgement of what the traffic would bear! However, now the company is under constant public scrutiny and detailed control by various government agencies. The highest standards of service are demanded, and, at the same time, the utmost economy is expected. Indeed, few countries enjoy better telephone service than that provided Canadians by Bell Canada. Yet, telephone rates have increased much more slowly than inflation and in some instances Long Distance rates have even decreased. How was the present somewhat involved system of discounts of 1/3, 1/2, 2/3, 3/4 at various periods of certain days decided upon? In a large measure - by using statistical analysis and mathematical modelling. A study was made of the number of long distance calls on different days and at various times. The effect of past changes in rates on the pattern of telephone use was studied. A prediction, using "time series analysis", was made of the probable increase in the use of the telephone in the immediate future. Such factors, and many others, were used as input for a mathematical model which predicted what net income would result if various rate structures were adopted. The model also tried to answer the question - what rate structure would encourage the most efficient use of the total long-distance network?

Mathematics - above all, statistics - is used by Bell Canada for many other purposes than deciding rates. An unexpected use is the following. If you were the manager of the Service Department of a telephone company, how much of which kinds of equipment would you normally stock in each Service Truck? Ten of everything? You would need a truck the size of a moving van! From a statistical study of the frequency of various types of service calls in a given district, a basic inventory for the service trucks has been decided upon which enables 95% of them to get through a normal day's work without having to return to the central depot for materials. Before 1940, Bell Canada employed 1 statistician. In 1960 the unit of the company for statistics, operations research and mathematical modelling contained 5 persons with mathematical training, 15 in 1965, 60 in 1970, and over 80 in 1975.

The Department of Health and Welfare disposes of over six billion dollars each year. At the Ottawa Seminar, D. Bray reported on some of the mathematical activities of the Health Protection Branch relating to testing drug products, conducting surveys such as the large study of nutrition, estimating microbiological hazards of imported foods and modelling health care delivery systems. It is the policy of the Department to scatter mathematicians and statisticians in small nuclear groups of three to five persons "where they will do the most good". Currently, there are 12 in the Health Protection Branch - some with PhD's in statistics and some with PhD's in biology with a strong statistics background. The number is expected to grow to 30 in a couple of years. In the other health programs, there are about a dozen mathematicians but the number is expected to increase.

The Department of Energy, Mines and Resources has recently created a new Canada Centre for Remote Sensing. Possibly, the layman can gain an understanding of the function of the Centre by recalling the extraordinary pictures of the surface of Mars and Venus which men and women of this generation have been priviliged to see. These pictures were transmitted electronically by means of an algebraically-designed code which was unscrambled by a computer. The design of the equipment and the theory of electromagnetic radiation used in transmitting the messages both involved mathematics (Boolean algebra, differential equations, integral transforms, etc.) of a level which would normally be understood only by a PhD in mathematical analysis or logic.

Now the Centre for Remote Sensing in Ottawa receives similar information from the Earth Resources Technical Satellite and seeks to use it for making maps of the whole of Canada for the study of changes in the environment - for example, of crops in the North. A normal image is represented in a code by over 30 million numbers. Ten to fifteen such images must be processed each day. As Dr. Bray points out, "The properties of these images are far from ideal. To analyze them requires real statisticians, not mere technicians who know how to apply the standard tests and techniques, because standard techniques just won't work." In 1973, there were no mathematicians in the group. By mid-1974 there were six. More will be needed.

At a Seminar held at Sault Ste. Marie, generously hosted by the Algoma Steel Corporation, interesting papers were presented by representatives of the Abitibi Paper Company, the Algoma Steel Corporation, Falconbridge Nickel Mines, International Nickel Company of Canada and Kimberly-Clark of Canada. All dealt with the application of the mathematical sciences in their company, most of which had been made possible only by the advent of the electronic computer. Chiefly, they involved statistical methods, operations research, computing, stimulation techniques and mathematical modelling. D.J. Dickson, Supervisor of Industrial Engineering for the Ore Division of Algoma, stated that Algoma faced very strong competition from open pit operations with higher grade ore - and that the company survived as an economic unit largely because of the efficiency of management which has turned to a variety of new techniques including the use of mathematics to solve day-to-day operation problems. In the past ten years, since computer facilities were made available on a regular basis, projects involving regression analysis, linear programming, network analysis, cost models and simulation have been completed. The use of simple or sophisticated routines is limited only by the imagination, knowledge and ability of the individual. These mathematical methods have been employed in the Department of Ironmaking, on a Study of Hot Metal Ladle Movement; and the Department of Operations Planning, to study the Strip Mill capacity of Utilities and to Optimize Energy Costs. In a similar vein, M.H. Kretzschmann of Falconbridge described the role - small but crucial - of mathematical methods in studying strain-stress distribution in the walls and rocks surrounding mine workings; heat exchange in underground mines; heat transfer and chemical processes in rotary kilns, etc. D.L. Riefstahl reported that Kimberly-Clark employs critical-path scheduling techniques to allocate the available manpower by trade to several interdependent maintenance projects.

These examples are surely sufficient to suggest that mathematics plays a crucial role in the conduct of contemporary Canadian business, industry, and government. Instances, analogous to those described above, were multiplied many times over in reports during the Seminars and in briefs to the Study concerning transportation, communications, finance, insurance, military strategy, industrial research and many other fields.

What Kinds of Mathematics?

Much of the mathematics currently being used in business and government is relatively simple. Indeed, a PhD in pure mathematics would be tempted to dismiss it all as "trivial" and of little interest. However, we advance the following hypothesis, for possible verification by some future Mathematics Study (Mark 7, perhaps!). "The main factors in determining the sophistication of mathematical techniques employed to solve problems in business or government are (i) the knowledge and ability of the mathematical practitioner (engineer, M.B.A., statistician, etc.) who has to solve them; and (ii) the capability of managers to appreciate the solution.

In Chapter II, rather oversimplifying, we summed up the recent social impact of mathematics in the three words: statistics, modelling, computers. The results of the Questionnaire reported in Appendix I supported our choice of these as the key words.

But each of them opens up large and complex vistas. The word "statistics" can suggest a simple-minded linear regression arising from product control in a nylon plant; queuing theory of some difficulty applied to the design of telephone switching machines; a highly sophisticated time-series analysis applied to the output of an electroencephalogram in an attempt to diagnose brain-tumours; or auto-regression techniques used in filtering out the "noise" in signals received by aircraft engaged in geophysical exploration by means of sensitive magnetic devices. Other statistical methods which were reported to the Study as currently being used include Factor Analysis, Sampling Theory, Multiple and Polynomial Regression, Significance Testing, Analysis of Variance. It is therefore apparent that nearly every aspect of statistical theory, which is commonly taught in Honours university programs up through the Master's level, has found significant application in Canadian industry and government. And not only the standard textbook material, for recall the warning of I. Crain at the Ottawa Seminar: the work of the Canada Centre for Remote Sensing needs mathematicians who realize that the standard techniques are insufficient to resolve the problems which the Centre is tackling. This remark was echoed by others in Agriculture Canada and in the Department of Energy, Mines and Resources.

The term <u>mathematical modelling</u> has become very much an "in" word. So much so, that in some quarters it seems almost indecent to deal with an economic or social problem unless it appears cloaked in a mathematical model. Indeed, a young professor of economics, extremely able mathematically, stated that no paper in a learned economics journal is now taken seriously unless it contains a lot of mathematics - and this, whether or not the mathematics is relevant to the topic at hand! Much engineering and the whole of theoretical physics is a large exercise in mathematical modelling. Maxwell's equations for the electromagnetic field can be used to predict the behaviour of radio and radar signals to greater accuracy than 1 part in 1,000,000. Dirac's model of the hydrogen atom predicts the spectrum of hydrogen to a similar accuracy. This model, modified by Bethe and Salpeter, is accurate to 1 part in 100,000,000. Another physical model of astonishing accuracy is Newton's theory of the earth-moon system which made possible the landing of men on the moon within a few hundred yards of the predetermined target. These physical theories employ ordinary and partial differential equations, integral equations, Fourier series, Laplace and Fourier transforms, and operators on Hilbert space. In recent years, physics has used such esoteric subjects as von Neumann algebras to describe the states of quantum systems and algebraic topology to classify Feynmann integrals. Indeed, in the Soviet Union, there is now a recognized scientific discipline called Algebraic Physics!

"Everyone talks about the weather, but no one does anything about it." Few topics seems to interest mankind more than the weather. Yet thirty years ago no one would have taken seriously the idea that we would ever be in a position to model the earth's atmosphere. The basic partial differential equations governing the motion of fluids have been known for a hundred years or more. However, the amount of data needed to specify the boundary conditions is so enormous and the problem of solving the equations is so involved, that even though a reasonable meteorological model could have been proposed, everyone knew it would never be of practical use. Now, with the help of a huge computer, owned by the U.S. government, into which is fed electronically masses of data from weather satellites and other stations, each day the model predicts the weather for the immediate future four times each day. It also disgorges long-range forecasts.

Perhaps the most newsworthy mathematical model in the past three years was World Dynamics of Jay Forrester and Denis Meadows which, employing the variables - Pollution, Population, Natural Resources, Capital Investment, and Quality of Life - purported to predict that the population of the world would rise to 16 billion by the year 2030 and then, as a result of widespread catastrophes, 80% of the earth's inhabitants would die in less than 30 years. This model employs rather simple mathematics. It has been criticized on this ground but even more trenchantly from the political and economic points-of-view. The Forrester-Meadows project is one of the more dramatic examples of a type of mathematical modelling of social and economic phenomena which is becoming increasingly popular. Hitherto, the kinds of mathematics commonly used in the social sciences include linear algebra (vectors and matrices), graph theory and mathematical programming, and elementary statistics (especially regression analysis and sampling theory). In addition, some ideas from calculus are conceptually indispensable, notably the interpretation of the derivative as a marginal rate in economics.

However, there can be little doubt that if there were a supply of persons who were sufficiently competent mathematically and fully steeped in the terminology and know-how of another field such as economics, psychology or physiology, many mathematical models of much greater practical utility could be devised. A striking example of this is the application by E.C. Zeeman of Rene Thom's Theory of Catastrophe to construct a model for the electrical activity in nerves. Thom's theory, based on differential topology of a highly sophisticated nature, provides powerful new concepts for discussing situations in which a smoothly developing phenomenon suddenly (catastrophically!) switches into a quite different mode - such as the development of an embryo, the onset of a prison riot, the activation of a nerve synapse. Using Thom's ideas, Zeeman simplified a previous model of nerve impulses involving ten empirically determined differential equations which gave more accurate predictions.

If one looks at the cover of Mathematical Reviews which lists the sub-divisions into which mathematics is currently partitioned, it will be seen that all the major categories and many of the sub-categories have been mentioned above as being currently employed in Canadian business, industry or government. Recall that in East Germany, category theory - perhaps the most abstract part of abstract algebra - has proved useful in designing a machine. One must therefore be convinced that - given the proper conjunction of real world problems, a resolute will to solve them on the part of management, and a team of imaginative persons familiar with the topic and having a bold wide-ranging mathematical competence - business, industry and government in Canada could profit enormously in many ways hitherto undreamt of by an effective exploitation of the mathematical sciences.

Consciousness-Raising: Two-Way Street

At the conclusion of the Seminar in Ottawa on Mathematics and Policy Planning, the Chairman emphasized the fact that mathematics is on the up-and-up because of the advent of the computer and because more people in different areas of society know that they need mathematics. Recall that Whitehead remarked on the paradox that this purely logical aspect of human beings, as it gets more and more abstract, also becomes increasingly useful and essential for controlling concrete phenomena.

At the Seminar, W.E. Krause, from the Post Office mentioned that mathematics is at most 10% of the problem in policy analysis. He is doubtless correct. However, it is a crucial 10%, and unless you have people of critical minds who know what is nonsense and will say so, the whole process of policy analysis becomes an exercise in futility. Some of the publicity accruing to the Forrester-Meadows model has raised expectations which cannot be fulfilled, and that is bad for everyone - bad for politicians who get seduced into relying on highly questionable methods, and bad for systems analysis because people get disenchanted.

The type of mathematics currently employed in policy analysis is often constrained, because the people who understand the problems know no mathematics other than addition and multiplication - so they add and multiply and use linear functions or polynomials. But there is a vast corpus of mathematics which is not being used. We need a general consciousness-raising so that people in government, industry and university will see possibilities of interaction and application which they did not see before.

You can say that this will happen inevitably, that it is the stream of history. One can read it in Morris Kline's fascinating book Mathematics in Western Culture. However, "There is a tide in the affairs of men, Which, taken at the flood, leads on to fortune". (Shakespeare, Julius Caesar, iv.3, 218). There is no doubt about the tide. Will we take it at the flood?

At the Seminar in Sault Ste. Marie, D.J. Dickson emphasized that what we are calling "consciousness-raising" is needed both on the part of management and university professors. He said in part:

> "Over the past few years I have attended conferences where the Academic community was brought together with the mining industry. Almost invariably the papers were divided into two groups which represent the theoretical and operations points of view. This usually results in each group knowing that the other had said something valuable, but not being sure what it was. Furthermore, it has resulted in the organization of conferences where only papers dealing with 'real' working systems were presented. While these have a definite place in spreading information, they do emphasize the gap between universities and industry.
>
> "I had the opportunity to return to university after having some work experience. I was fortunate that the course had several lecturers with considerable industrial experience. These people had the ability to relate the theoretical to the practical, explaining the limitations and simplifications that were involved. As a result, this was a most productive year in terms of personal satisfaction and increased ability to perform my job. Perhaps this is one pattern for advanced mathematical education to follow.

"I think it is fair to say that the Algoma Steel Corporation has had considerable success with acceptance of mathematical results produced by computers. Some of this may be attributed to awe of the 'electronic brain', but mainly it is the result of people participating in the solution of their problems. In order to get this cooperation it is necessary to demonstrate a reasonable possibility of success. This can be done by first solving the seemingly insignificant problems which 'bug' people. Once credibility has been established, then the larger problems requiring a lot of mutual confidence can be attempted. Perhaps similar cooperative effort between universities and industry is required to solve the problems we are discussing today.

"I am encouraged that this study is taking place. I hope the resulting recommendations will have realistic goals in terms of what can be accomplished, both in the short term and over the long run."

In a similar vein, Dr. D.R. Miller, Head of the Biomathematics Group of the NRC, addressed himself to the difficulties of trying to apply mathematical techniques to very complicated systems, as inevitably happens to problems involving ecology, regional planning, pollution, or energy supply. The blame for some rather disappointing results must be shared almost equally by the mathematicians who work on such projects and the non-mathematicians who direct them.

On the one hand, very few mathematicians working in this area are following the basic philosophy of science, which is to inductively frame a hypothesis, to deductively work out the consequences in a form that can be tested, to experimentally try to verify or disprove the hypothesis, and then to return to frame a new hypothesis (if necessary) on the basis of the experimental evidence. The journals are full of mathematical models based on a priori assumptions which the author has no intention of testing. Frequently, they are trivial. Such "work" seriously damages the credibility of the mathematical modelling profession.

On the other hand, those who administer such projects very often do not use the mathematical talent that is available to them. The mathematician is frequently asked to provide services of a purely technical nature, long after the significant details of the project have been thoroughly worked out. In every other field of science, the directors of multidisciplinary projects assume that research in methodology and technique will be carried out as part of the work. The same should be true in mathematics: time and resources should be available from the very outset to allow for the formulation and testing of several successive mathematical descriptions.

This, of course, has important implications for academic programs in mathematics at the undergraduate and graduate levels. Concerned as they are with technique, university educators have allowed the formulation and testing of mathematical descriptions to become much too downplayed, so that it forms far, far too small a part of the curriculum.

Positive Suggestions

What concrete action can be taken? From the Seminars and briefs has emerged a wild welter of positive ideas. We record here, in somewhat telegraphic style some of the suggestions which seem most relevant to the subject of the present chapter. Some of them were put forward by one individual, some represent a consensus of a significant group. But they are advanced here merely on their own merit for consideration and possible eventual action by those many Canadians who are concerned for mathematics in Canada.

(i) Mathematics in other fields

-Cooperative research ventures should be encouraged involving university mathematicians on the one hand and government departments or industry on the other.

-Every research project or model-building exercise in which mathematics has a significant role should involve a mathematician from beginning to end.

-Mathematicians engaged in model-building must be given time and funds to test, adjust, and retest their models several times.

-More attempts should be made to apply non-quantitative methods (graph theory, topology, boolean algebra, metagame theory) to problems in the social sciences, in the biological sciences, and in policy analysis.

-Managers should be involved in operational research activities from beginning to end.

-Problems of data storage, data retrieval, data linkages, and data display are not given the attention they deserve.

-Statistical consultants in government and industry should be spread out in the organization where they can do the most good.

-Mathematicians wishing to work in government and industry should be prepared to function as peers in very large multidisciplinary teams.

-Some understanding of mathematical methods and their applications should be imparted to present and future managers by means of short courses aimed at developing mathematical literacy. Managers would be less frightened by mathematics and computers if they were more familiar with the jargon.

(ii) Liaison

-Committees and special seminars should be established, involving mathematicians from universities, government offices and industrial concerns to promote interaction.

-Academic mathematicians should be encouraged to spend leaves of absence or sabbaticals in industry, government or other university science departments.

-Visitors from other university departments, government and industry should be invited to give seminars or workshops in Mathematics Departments.

-Special short courses should be provided to upgrade the mathematical skills of employees in government and industry.

-Mathematics professors should become more involved in consulting and problem solving activities both inside and outside the university.

-PhD's in mathematics should broaden their outlook by working in other milieux before taking a permanent position in a Mathematics Department.

-A list of agencies which would welcome mathematical interaction should be assembled and publicized.

-Professionals in business and government could serve on Advisory Councils to help universities develop new, or to modify continuing, programs.

-Interested mathematicians should take advantage of the NRC Industrial Postdoctoral Fellowships and Senior Industrial Fellowships.

-People with strong mathematical backgrounds, but whose primary training is in another field, should be recruited as full-time faculty members in Mathematics Departments.

(iii) University Education

-Mathematics curricula should provide some experience with all aspects of problem solving, from initial formulation to final interpretation, including the revision and testing of successive mathematical models.

-Undergraduate and graduate mathematics programs in the Mathematical Sciences should be diversified to encourage students to become knowledgeable in at least one field of application.

-The need for mathematical skills in government and industry should be publicized in a responsible way within the universities.

-Relevant short-term work experience (through cooperative or summer programs) should be made available to mathematics students interested in non-academic employment.

-In view of the enormous impact which computers have had, Mathematics Departments should become more closely affiliated with Computer Science Departments.

-Intermediate-level undergraduate and Master's programs in mathematics or statistics could involve the writing of theses based on practical problems arising in government or industry. Individuals outside the university should feel greater responsibility than at present to suggest such problems and provide the possibility for their study.

-Many mathematicians do not perceive the value of other disciplines within a mathematics education. Science and Commerce departments should be encouraged to offer courses of interest to mathematics-majors to give them the experience of being a "specialist" in a team working on ill-defined problems, and learning to communicate with "non-specialists".

-Students should be encouraged to obtain multidisciplinary training - such as a bachelor's in mathematics followed by a master's in economics, or a bachelor's in engineering followed by a master's in mathematics.

-Undergraduate and graduate courses in mathematical statistics should attempt to provide students with some practical consulting experience.

-Imaginative and attractive programs in the applications of mathematics (e.g. engineering mathematics, bio-mathematics, policy analysis) should be devised. There could be Master's and Doctoral programs in Industrial Mathematics.

(iv) Incentives and Mechanisms

-Consulting activities and interdisciplinary work must be given more recognition by promotion and tenure committees in the universities. Deans and Department Chairmen must take a strong lead in this.

-More incentive should be provided for Mathematics Departments to produce employable graduates at all levels, even at the expense of research productivity.

-Granting agencies should finance interdisciplinary work on a team basis rather than on an individual basis.

-A clearing-house should be established to put users of mathematics in touch with mathematicians who are willing to do consulting work and advise universities when people from industry and government might be available as guest lecturers.

-Sabbaticals and leaves of absence should be provided which would allow professors of mathematics to become conversant with other fields. They may not solve all - or any - of the problems of industry and government. But they should return to university with a clearer idea of the reality which their students will confront.

-A list of mathematical problems culled from government and industry in Canada should be circulated on a regular basis to Mathematics Departments.

-A collection of solved problems from various fields of application in Canada should be made available in book form for the benefit of mathematics students and faculty.

-Papers of a mathematical nature which appear in applied journals should be given more weight by the granting agencies.

-An Applied Mathematics Institute [3], possibly with provincial units, should be set up to provide services for users of mathematics.

Notes - Chapter III

1. "Bud" Drury is one of the most influential ministers in the Cabinet. The quotation is from his speech which opened the series of three Seminars organized in Ottawa on the role of Mathematics in the Federal Government. He was then President of the Treasury Board but subsequently became the Minister of State for Science and Technology and also the Minister of Public Works.

2. The reference here is to a recent theory of Rene Thom which is described later in this chapter.

3. An entire subsequent chapter discusses this proposal.

Chapter IV

Mathematics Teaching in the School

"... it is incumbent on our scholars to learn the rudiments of mathematics and astronomy." - Tz'u-hsu, Empress Dowager of China[1]

"The final analysis of the effectiveness of any mathematics education reforms rests in considering the question: 'What is the position of mathematics education in advancing national goals?' ... Chinese educational reforms in mathematics have fitted successfully within the plans for national development. ... What is important at this time is that as much of the Chinese society as possible achieves some appreciation of the power of mathematics so that this generation of worker-peasants, having been thoroughly exposed to the broad aspects of mathematics, will bring forth a generation intent on expanding this knowledge. Then a whole society, rather than a select few, will advance in mathematical potential, and through it the industrial potential of the People's Republic of China will become most formidable." Frank Swetz[2]

Almost as soon as children speak a few words, their eager parents start teaching them to count. It is always a moment of great joy and pride when the child is able to count confidently up to ten. But rote counting is far - several years - from the moment of appreciating that three apples and three toys have something - threeness - in common. This is the moment when the child becomes a mathematician! If "man" is defined as a "rational animal", then perhaps it is no exaggeration to assert that man was truly and assuredly man when for the first time he intuited, however dimly and inarticulately the idea of the <u>cardinality</u> of a set. That moment marked an extraordinary breakthrough in man's ability of abstraction and resulted in a remarkable new power to understand and control his environment. Perhaps within a few tens of thousands of years of that event, man abstracted the notion of <u>serial order</u> and of simple geometrical figures such as circles or triangles. But it took many millenia, perhaps millions of years, until Pythagoras, Euclid and their colleagues achieved the art of deductive reasoning and the possibility of analyzing the complicated properties of geometrical figures which are now comparatively easy for a student in Grade 10. Between Grades one and ten, a child recapitulates millions of years of the intellectual saga of the human race and, in so doing, acquires mental tools which are essential to the existence of modern society. Mathematics teaching in the elementary and secondary schools and in community colleges plays a crucial role in determining the quality of civilization. Certainly it is a critical factor in enabling a country to compete economically on equal terms with other technolgically advanced nations.

The quality of mathematics instruction in the schools is undoubtedly the most important single factor affecting the health and vigour of the whole mathematical ecosystem. However, this, in turn, depends critically upon the mathematical life of the universities which will be strong only if the professors are keenly interested in teaching and at the same time are actively attempting to extend their own knowledge. Since research is the elan vital of the mathematical enterprise, a public policy which fails to encourage mathematical research or attempts to dissociate it from teaching is short-sighted and in the long-run will be recognized as misguided.

No subject was discussed at greater length by contributors to the Study than mathematics teaching. No one was satisfied with the current situation in Canadian schools.

A Mathematical Malaise

There is a large body of informed opinion that the teaching of mathematics in the elementary and secondary schools in Canada is unsatisfactory. The responsibililty for this devolves directly and immediately upon the Ministers of Education and the provincial governments. However, ultimately, responsibility must be assumed by the whole mathematical community, and notably by university professors. The situation varies from province to province and from school to school.

The most convincing objective piece of information which was presented to the Mathematics Study consisted of a Report on the Testing of Arithmetic issued by the Department of Education of British Columbia. The Stanford Arithmetic Test was administered in 1964 to 99%, and in 1970 to 95% of Grade VII pupils in British Columbia. The test consisted of two parts referred to as "Reasoning" and "Computation". The scores obtained by pupils in U.S. schools in

1964 were used as the fixed standard against which the B.C. pupils were measured in 1964 and 1970. In 1964 the B.C. pupils were ahead of their U.S. counterparts by 18 months in reasoning and 11 months in computation; in 1970, by 8 months and -1 month respectively. Thus the median performance of the B.C. pupils had deteriorated markedly in the six years from 1964 to 1970. There were rumours of a similar development among their U.S. counterparts, but no hard data was available. Interestingly enough, the performance of the top percentile of B.C. pupils had hardly changed. It was the performance of the future "average citizen" which had deteriorated.

The situation in British Columbia is not exceptional. In fact the overall picture in Canada at present contains so much distress, unease and confusion that energetic steps must be initiated immediately to improve the situation. This opinion was expressed in many briefs submitted to the Mathematics Study and reiterated ad nauseam by everyone we consulted. The only exception was an occasional overly sensitive offical of a Teacher's Federation or Department of Education who felt threatened and struck a rigid defensive stance. In order to effect significant improvement, such steps must be pursued consistently for ten to fifteen years - uninterrupted by fadism of changing bureaucrats.

"Misery likes company" so possibly there is some consolation in the fact that, apparently, Canada is not alone. At the three sessions of the International Commission for Mathematics Instruction, which took place at the University of British Columbia in August, 1974, delegates from one country after another reported significant and trying problems in mathematics teaching at the elementary level. In Poland, all 30,000 school teachers in Grades I-III were expected to watch a weekly program starring Professor Z. Semadeni and to submit regularly the solutions to a series of simple exercises. The Course, which began in January 1975, attempted to overcome their fear and lack of understanding of basic mathematical ideas.

In 1964 a study [3] of mathematics skills of school children in twelve countries was conducted under the auspices of UNESCO. The study involved 132,775 students and 13,360 teachers. The 13 year olds of Japan and Belgium ranked first and second in the evaluation while those of the USA and Sweden were eleventh and twelfth. The test-scores, with standard deviations for these four countries and the average for all twelve countries were as follows.

	Score	S.D.
Japan	31.2	16.9
Belgium	27.7	15.0
U.S.A.	16.2	13.3
Sweden	15.7	10.8
Average	19.3	14.9

Even though the Japanese students excelled in this test, Professor Iyenaga reported at Vancouver that there is widespread criticism of mathematics teaching in the Japanese schools.

A common method of evading the real issues is to indulge in vituperation of the "New Maths" and the monstrous abstract mathematicians who foisted these dreadful irrelevancies upon us! Undoubtedly, there have been massive absurdities perpetrated in elementary and secondary schools of Canada and the USA and in a number of other countries under the banner of the "New Mathematics". However, we should not conclude that the basic aims of the mathematical reformers of the late 1950's were improper nor that radical reforms are unnecessary. Rather, educational reform in so fundamental a subject as mathematics can be achieved only by consistent effort over a long period of time. Much of the damage has resulted from the eagerness of publishers to capture the market and quickly issue books by hack writers which emphasize the use of new jargon but are unsuccessful in grappling with ideas.

How shall we recoup what Morris Kline [4] and others luridly think of as the debacle of the New Maths? Certainly the answer is not to return to rote memorization of number facts. Indeed, not to return anywhere, for the old program in mathematics instruction in Canadian elementary and secondary schools was completely inadequate for a modern society. It may have been adequate in the 1920's but certainly not today.

A Primary Aim for Mathematics Education

In 1957, a distinguished Swiss banker who had been educated in the years 1885 to 1895 looked back on his mathematics education. Even though he was in his eighties, there was deep trauma in his tone as he recalled struggling with "The rule of three" - "l'infecte regle de trois". (Given any three of a, b, c, d, if you know that a:b c:d, find the fourth.) This was the most sophisticated mathematics he had learned. It was sufficient for a career in which he became president of the Swiss equivalent of the Bank of Canada and one of Switzerland's financial representatives at the Versailles Conference. Today, his counterpart must feel comfortable in talking to computer scientists and systems analysts. He must have an informed view of what weight he should attach to the output of a variety of complex mathematical models of the national and world economy. This does not mean that he has to be a computer scientist and a systems analyst. It does mean that he needs more than a rote knowledge of the number facts and the rule of three! He needs an understanding of what mathematics is, what you can expect of it and - perhaps more important - what you cannot expect of it.

The mathematical needs of a bank president certainly exceed those of average citizens. However, unless the latter know more about mathematics than the number facts and the rule of three, they will feel alienated from the modern world. The only way to obviate such alienation is to ensure that there is universal knowledge of what mathematics is. The notion that mathematics is an abstract and exotic doctrine whose mysteries can be penetrated only by a select group of initiates must be banished from Canada. If it persists, the majority of people will increasingly feel themselves helpless pawns in an impersonal technological society controlled by the mathematical high-priests of systems analysis, operations research and electronic systems.

Marxists and economists frequently argue that the most important factors affecting the development of nations are economic. However, it may well be that a more significant change in western civilization is illustrated by the fact that whereas in 1900 a rote knowledge of some arithmetic facts was all that average persons needed, now they cannot relate comfortably to their environment unless they understand what science and technology are all about and this requires a knowledge of what mathematics is about. This change - concerned as it is with man's understanding of his place in his world - has almost a religious dimension.

We, therefore, propose as the chief and overriding aim for the teaching of mathematics in the schools <u>the widest possible dissemination of an understanding of what mathematics is and what it is not</u>. As far as we know, this has never been adopted as a conscious aim of any school system. When it is achieved, the average citizen will understand that mathematics is (a) <u>a way of thinking</u> which provides a powerful tool for analyzing subtle and unobvious aspects of our experience, and (b) a <u>cultural resource</u> which can add interest and enjoyment to life. Further, (c) it will be apparent that the symbolism of mathematics - algebraic and graphical - constitute an important language which is essential for communication of ideas and for the formulation of societal goals.

Obstacles

As far as we are aware, there has never been an explicit attempt in any large school system to achieve the above aim. Rather, the curriculum has grown in a haphazard manner in response to the simplest practical demands of society and the whims of textbook writers. Normally, when a Ministry of Education decides to undertake a revision of the curriculum in any subject, the responsibility for proposing changes is given to a committee dominated by practicing teachers since they are presumed to know what teachers are capable of teaching and what pupils are capable of learning. Such factors, should, of course, be given due consideration. However, the imagination of each individual in such a committee is usually narrowly constrained by his or her particular conditioning so it is very difficult to introduce basic changes into a school curriculum. There are many other reasons for this. Here we shall describe four.

(1) Emotion versus Reason

Mathematics does not come as naturally to mankind as eating or love-making. Shakespeare's phrase "all sicklied o'er by the pale cast of thought" conveys the feeling of most of us about logical reasoning. Logic destroys the pulsing energy of vivid emotions which give meaning and richness to life. Surely, a chief source of difficulty in the teaching and apprehension of mathematics is the fact that man is controlled almost wholly by his emotions, and that the power to reason and the capability of abstraction are a relatively recent and therefore superficial achievement of the evolutionary development.

Even so, our logical powers, which have their chief employment in mathematics, are an essential element in distinguishing mankind from animals. In a recent article, David Wheeler has argued[5] that our thought processes are actually mathematical.

"The special mental functioning that I call mathematization may be operating even when there is no recognizable mathematical product coming out of the activity. I am not sure that Boole[6] was the first to propose that the natural operations of the mind are algebraic in character; I am sure that this is what he meant and that it is unfortunate that his discovery has been debased, for many people, into the theorem that two-valued sentence logic is isomorphic to an algebra of sets. Much more recently, the Swiss epistemologist Piaget has asserted that the structures of mental operations are isomorphic to the 'mother structures' used as foundations by Bourbaki. The evidence for an algebra of the mind is even easier to find than in these two sources.

"It has been repeatedly stressed by observers that within a very short time of beginning to speak, young children utter grammatically correct phrases and sentences that are not copies of any that they have heard others speak. It is clear that they could not operate autonomously within the grammar of the language without the capacity to handle classes (nouns, verbs), inclusion and intersection (adjectives, adverbs), relationships (prepositions), transformations (tenses) and substitutions (pronouns). I mention only the simplest and most obvious features: in fact the transformational requirements of a confident use of grammar are very complex and still defy complete analysis. But the chief points at issue are that (a) the child's mastery of grammar can only be adequately described in terms of mathematical operations, and (b) this mastery is not derived by imitation. The algebraic character of mental functioning seems proved by this one example even though the evidence can be multiplied by considering other functionings - visual perception, for instance."

The chief and very important point of Wheeler's article is contained in the opening sentence "It is more useful to know how to mathematize than to know a lot of mathematics". He claims that too great an emphasis has been placed, in the traditional approach to teaching mathematics, on instilling facts and theorems from outside - discovered by others. He admits that the "learning" of parts of the traditional material is useful, even essential. However, the absorption and retention of "facts" should not be the main object of the teaching-learning process - but rather an important by-product. The main aim should be that of exploiting, and extending the ability to "mathematize" which is inherent in all thinking individuals - by encouraging the student to exult in the "algebraic" character of mental functioning.

Probably, it will be widely agreed that "mathematizing" is a function of the post-mammalian[7] part of the human brain and therefore inextricably intertwined with the faculties which distinguish humans from other animals. The limbic and reptilian aspects of our brain are far from being controlled by reason. In a real sense, they are threatened by and rebel against reason. Thus, to mathematize involves struggle. But it is the struggle intimately involved in, and integral to, the attempt of humans to assert their humanity over their animality.

(2) Undercurrents of Anxiety

The Study forced vividly on our attention the fact that a deplorable attitude to mathematics is suffused throughout our society. This attitude creates unfortunate mental blocks in nearly everyone, particularly in managerial and decision-making personnel. It is a compound of a somewhat envious admiration of those, supposedly few, who are gifted with mathematical ability and a pooh-poohing of mathematics as an airy-fairy irrelevance to "real-life" considerations.

We fix on the attitudes of elementary school teachers as the strategic factor in propagating this unfortunate attitude in Canadian society and, hopefully, in correcting it. However, we do not "blame" elementary teachers for picking up an attitude which is endemic to our society. Several years ago a study revealed that the vast majority of elementary school teachers in the U.S.A. have anxiety feelings about mathematics. Our discussion with mathematics instructors in Universities and Training Colleges in Canada leaves us in no doubt that the same holds true of Canadian teachers. For example[8], of 92 elementary teachers who pursued a course in 1972 to up-grade their status, in deciding which four of five options they would take, 85 decided to avoid the mathematics course. Personal biographies which they all submitted confirmed the impression that they were, indeed, "avoiding" mathematics because they disliked or feared it.

It is commonly admitted that young children and even animals can sense feelings without verbal communication. So it is not surprising that many children living in a situation suffused with a sense of unease about mathematics unconciously absorb this debilitating attitude.

(3) Inadequate Training

The preceding two factors have been operative over many years and not only in Canada. As a result, it has been impossible for past programs for training teachers to ensure that the majority of elementary teachers have a confident understanding of the mathematics which they are called upon to teach. Dr. C.E. Beeby[9] has asserted that any educational system is in one of the following four phases. I) The Dame School Stage, in which teachers are neither educated nor trained. II) The Stage of Formalism, in which teachers are trained but poorly educated, the classroom organization is highly organized and the syllabus is rigid. III) The Stage of Transition, in which teachers are trained and somewhat better educated, but lack full professional competence. IV) The Stage of Meaning, in which teachers are well- trained and well-educated, meaning and understanding are now stressed in the schools and individual differences among students are taken into consideration.

It is our impression that most schools in Canada are still in Stage II as far as mathematics teaching is concerned, some are in Stage III, but very few have attained Stage IV. To make significant progress will require long, sustained - almost heroic - efforts of teacher training in colleges and university and an imaginatively conceived large-scale in-service training program. Such a program can be successful only if it attracts the enthusiastic effort of many lively, able university mathematicians.

(4) Textbook Situation

The economics of the publishing business in North America militates powerfully against the production of good textbooks or other teaching materials. Because what it means to mathematize is so little understood by curriculum planners, teachers, or editors of publishing houses, the textbook market is an easy prey to fadism. The publisher tries to sell his wares by a flashy use of colour, by transparency inserts, or by some extraordinary break-through such as teaching "dividing" by subtracting "sets" of cardinality equal to the divisor!

We cite two examples of the harm which results from the manner in which the textbook market functions in North America. And, of course, as far as the publishing of mathematics is concerned, Canada is a mere feather on the U.S. eagle.

Fifteen or twenty years ago there was a sudden fad of "programmed" learning of various varieties. The publishers feared and/or expected that this was the coming "thing" so many quickly produced huge series of programmed texts. A cursory glance at most of these convinces any critical informed reader that they were rushed into print by authors who may have had some knowledge of the format of linear or branched programming but essentially no real understanding of the subject-matter with which the text dealt. Currently,[10] Professor A.J. Gold of the University of Windsor is undertaking to develop remedial courses for Ist Year students who are deficient in the prerequisites for university mathematics courses. He searched diligently for a good programmed text treating elementary algebra. He found only one. It is out-of-print. The exaggerated oversell brought programmed-texts into disrepute. This was unfortunate since they can fulfill a useful role.

A publishing fiasco of much greater import for mathematics was triggered by the advent of the so-called "New Mathematics". Here again the U.S. publishers rushed in and flooded the market with books which appeared to meet the new desiderata. This was achieved by scattering the words "set", "commutative", "additive inverse", "associative" and "distributive" liberally throughout the text in various inappropriate and, by chance, occasionally appropriate places. Probably, the nonsense perpetrated in some of these books has been the greatest single factor in bringing the new approach into disrepute. Most of them attempt to teach jargon rather than ideas. Since, for the preceding three reasons, many teachers do not have a sure grasp of the ideas which the mathematical reformers are attempting to disseminate, few are able to make good the deficiencies of bad textbooks or even to convey the meaning implicit in the few good ones. Griffiths and Howson[11] quote Dr. Beeby to the effect "No change in practice, no change in the curriculum has any meaning unless the teacher understands it and accepts it". This truth clearly determines, in particular, what a teacher makes of the textbook.

In most parts of English-speaking Canada the textbooks for elementary and secondary mathematics programs come from the U.S. or are Canadianized versions of U.S. texts modified in mathematically trivial ways. There is an exception in Ontario where three publishers developed series of texts for a curriculum promulgated by the Department of Education which was based on extensive consultation among Ontario university professors, high school teachers and departmental officials under the aegis of the Ontario Mathematics Commission. These textbooks, which brought the "New Math" to Ontario, are quite conservative in comparison with many of their U.S., French or Belgian counterparts.

There is no agreement among Ontario mathematics professors as to whether the net effect of these changes was good or bad. Since the new curriculum has been accompanied by very great changes in attitudes among students, changes in their study-habits in an increasingly affluent society, and by the disappearance of Grade XIII examinations, it is not possible to isolate the effect of any one factor. At Queen's University throughout the past fifteen years the incoming freshman year has been roughly of constant ability and social structure. There, the subjective judgement of the mathematics professors is that the average freshmen from Ontario do not have quite as good command of the manipulative side of algebra and trigonometry as they had when the Grade XIII examination was operative. However, they are more than compensated for this by a greater ability to grasp mathematical ideas. A lamentable weakness under the old and the new regime was and is their lack of three-dimensional spatial imagination.

In French-speaking Canada, the textbook situation - though not dominated by the U.S. - is at least as difficult as in English Canada. From the elementary level through CEGEP there are few distinguished mathematics textbooks written by Quebecois. So, frequently the only ones available were written in France for the rigid, conservative and highly idiosyncratic traditional French program or the recently overly abstract texts by G. Papy in Belgium or by Bourbakists in France. However, it may well be that a dramatic improvement is in the making centered in the Tele-universite of the University of Quebec. In supporting this, the Department of Education of Quebec is showing more awareness of the need for energetic action to improve mathematics education than is currently manifest in any other Province.

The position of French-speaking students and teachers in Ontario is doubly difficult since they are attacking a curriculum for which few adequate textbooks or other teaching aids exist in French. A Sub-Committee of the Ontario Association for Mathematics Education, consisting of French-speaking teachers, has been attempting to produce some teaching aids, on a voluntary basis. However, an adequate approach to their problems must involve strong initiative from the Department of Education which has not been forthcoming. The Federal Government dispenses large sums to the Provinces for the support of

French. Surely a first claim of such funds should be the provision of adequate textbooks for the children in the Ontario Ecoles Francaises.

The Way Forward

Social attitudes such as the wide-spread fear of mathematics cannot be changed overnight. Furthermore, no educational system is better than the teachers in it. The education of a teacher takes five to ten years. To attempt to change the ideas and attitudes of practising teachers is a major enterprise. Most of them are so overburdened or committed to getting their students through the received curriculum that they have little interest, time or energy to participate enthusiastically in in-service training schemes. Like everyone else, they need the holidays to recoup physical energy and to relax in order to preserve their sanity. They cannot be expected to eagerly devote their holidays to attend courses, especially if they must pay for such courses themselves. That so many teachers do exactly that is a convincing measure of the unselfish commitment which pervades the Canadian teaching profession. There is no doubt that, on the whole, they will cooperate effectively with any responsible plan which is clearly in the general good.

Earlier in this chapter we quoted Dr. Beeby's "model" of educational systems. It is one of his assertions that it is impossible to omit any of the four stages in the development of an educational system. The many factors involved in the process of modifying curriculum in the schools are analyzed in detail by Griffiths and Howson to whom [11] we have already referred. Their book is strongly recommended as required reading by anyone concerned with the issues involved in making headway in improving mathematics education. It has been said - in jest, but half seriously - that when the Swedes decided on major changes in the schools, they deliberately planned on taking thirty years to effect them. Among other factors, it was required as necessary that a fair number of persons should die or reach retirement before a real change could be effected.

Possibly Canadian teachers and educational bureaucrats are more flexible than their Swedish counterparts. Nonetheless, it must be acknowledged at the outset that the type of changes needed in the teaching of mathematics in Canada will not be effected in one or two years, nor even within the life-span of one Government. Therefore, the enterprise needs to be widely understood and approved by the public, by various levels of government, by teachers' organizations and, of course, by teachers of mathematics in schools and universities. A consistent and well-directed effort will be needed over many years. Nor should it be subverted by the commonly observed phenomenon which dictates that each new Deputy Minister or Superintendent of Education must cancel all the programs of his predecessor in order to assert his own ego and prove to the world that he actually has arrived on the scene!

There are many ways forward for improving mathematics education in Canada. There is no doubt that the time is ripe to make a great leap forward in Canada. Evidence for this is the positive and widespread interest with which the Mathematics Study was received on almost every hand. Nearly everywhere the Study was recognized as most opportune. The fact that almost 300 individuals and groups from every Province submitted briefs bearing on all aspects of the role of mathematics in Canada convinces us that there is a large body of persons willing and competent to help in this enterprise. Their energy needs only to be channeled by means of sensible leadership.

However, one fact of overriding importance must be in the forefront of everyones' efforts. <u>To mathematize is to joy</u>! It is to exult in one of the powers distinctive of our humanity. It is to employ explicitly structures which are implicit in the higher levels of our brain. To convey a tone of confidence and satisfaction in doing mathematics should predominate over obsession about "covering" a predetermined curriculum. This should be true in teaching students. But at this juncture, it should be especially true in teaching teachers and prospective teachers. There is no other way of dissipating the widespread anxiety with which mathematics is regarded.

There are many constructive practical steps that can be taken in Canada at this time. Here are four.

Use Consultants

Much greater use could be made of "Master" Teachers or Mathematical Consultants throughout the school system from kindergarten through university. If there are specialists for French in Grade II in English Canada, why should there not be a system of mathematics specialists or, at least consultants, from kindergarten?

The well- known poet, W.H. Auden, asserted that well into their teens, children should study only two subjects: their native language and mathematics. These two are essential for communication between humans. Most other subjects with which the curriculum is cluttered are largely non-essential or require relatively mature experience before they can have existential meaning. In view of much of the permissiveness which dominates the up-bringing of young people in Canada, the political implications of attempting to implement Auden's view are horrendous. However, the strength of his position is sufficient to require that at all levels of the Canadian educational system we should make every possible effort to ensure that the youth of our society gain a mastery of French or English and a confident control of mathematical thinking.

Mathematical consultants should be given authority to intervene actively when they deem a teaching situation is unsatisfactory. Their role should be more than advisory. Naturally, they must be tactful people who will not unnecessarily exacerbate the feelings of uncertainty about mathematics which are current among teachers and officials of Boards and Departments of Education.

In-Service Training

A comprehensive program of in-service training for teachers of mathematics should be instituted immediately involving the following elements:

(a) Summer courses organized by universities, by Provincial Departments of Education, by teachers' organizations, or modelled on the wide variety of Summer Institutes for college and school teachers sponsored during the past decade in the U.S.A. by the National Science Foundation.

(b) Universities could develop Correspondence Courses - both credit and non-credit courses - aimed at increasing the mathematical confidence and competence of teachers. School Boards should encourage their teachers to participate by defraying the costs involved and by providing released-time.

(c) The Professional Development Days organized in some provinces by teachers' organizations could be used much more effectively if a well-conceived cumulative program were designed and mathematical consultants and university or community college professors were available as resource persons.

(d) The facilities of ETV could be used to beam into the schools courses in which the teachers[12] would be encouraged to participate actively. We have already referred to the ambitious course offered to the teachers of Grades I-III in Poland. In principle, all 30,000 such teachers were obliged to participate. There were thirty one-half hour transmissions on the main TV channel at 4:00 p.m. during the period January-June 1975, with repetition in the late evening. Eight pages of exercises were published twice a month. The exercises are very simple and consist mostly of graphical schemes to be completed by the teacher. They were marked in each county by persons provided with a set of correct solutions. The goal of the course is to prepare teachers for a basic modernization of the mathematics curriculum. The whole project is directed by Professor Z. Semadeni, who is an able research mathematician and is also very sensitive to the human and political factors involved, so the project is one of the most significant efforts hitherto attempted to use TV for mathematics education.

(e) Groups of interested teachers in geographically convenient proximity - for example from two or three elementary and secondary schools - could form discussion groups to read books or short articles which would help them gain a deeper understanding of the role of mathematics in Canadian society.

However, we issue a strong warning against the erroneous idea that anything worthwhile along the above lines can be accomplished by Ministerial fiat supported merely by money. Money will certainly be needed. But the main need is dedicated personnel of competent and distinguished minds. Currently, these are in very short supply in Canada. An in-service program of the sort we are proposing will only succeed if a large number of enthusiastic university professors devote a great deal of time, energy and creative thought to it. They need not be paid extra (apart from out-of-pocket expenses, which should be made liberally available). This task is so important to the future of Canada that it should be viewed by Chairmen of Departments, Deans and University Presidents as being at least as important as the sort of research to which the majority of young mathematicians are currently devoting much of their time. It should be rewarded generously in prestige and promotion.

Such an in-service program is unlikely to succeed if it depends on purely voluntary participation by teachers. This is suggested by the experience at the University of Winnipeg in which only 7 of 92 elementary teachers, who enrolled in a program to up-grade their certificates, opted to take the mathematics course which was offered to them. Most teachers are frightened that they will not do well in a mathematics course. Their fear of possible embarrassment is an enormous hurdle. They will need to be encouraged by strong moral persuasion. Further, the organizers of courses must begin where the teachers are and seek to carry them forward with joy at all stages. Therefore, at the beginning, there should be no _a priori_ ideas about minimum content. For many years, the essential point will not be the _content_. It will be the _feeling_.

Role of University Professors

The mathematics training of future teachers needs to be rethought and greatly improved. There is a fetish among university professors in nearly all departments that the priceless gift, "university credit", must not be given for a course of which the content is not of "true university level". Thus, few English Departments can bring themselves to teach grammar or prose composition; the French Department plunges the naive Canadian student into the poetry of Racine or the sophisticated thought of Camus. The process could be illustrated by every discipline. The core of the university program - in mathematics as in most other subjects - was established for the benefit of honours students who might conceivably end up as creative mathematicians. It is this sequence which dominates the academic value scheme. Consequently, the "Pass" or "General" students are normally regarded as second-class citizens in Academia. Those who are not majoring in mathematics know before they start that they will never understand mathematics. They did not understand it in public or high school, and since the various programs in mathematics presuppose that students are positively motivated to mathematics and have at their finger-tips all that they learned in twelve years, they know with a powerful gut feeling that mathematics is not for them.

A first necessary step forward is for Departments of Mathematics to change their attitude to the problem of "mathematizing" the majority of students who will never become practising mathematicians. It is not adequate to sneer at "Cultural Mathematics" as a "bird" course which is not mathematics in any real sense. Of course, this can be a valid evaluation of many such courses offered in the past merely for the purpose of enabling a non-mathematical student to obtain a "credit" or to fulfill a "distribution requirement". However, it should be possible to create courses which will arouse an interest in and an understanding of mathematics for students who did not have this experience in school. Far from being "bird courses", if they engross the interest of the student, they could be as demanding as any honours course. To create such a course will require much greater effort than lecturing from Ahlfors "Complex Variables". Such a course will be dealing with ideas, shaping basic attitudes, leading the student into new intellectual territory, teaching him to feel at home with new concepts and new thought processes and therefore be much more worthy of a "university credit"than most Third and Fourth Year Honours Mathematics Courses which are largely technical regurgitations.

Textual Aids

A new approach must be developed to the production of mathematics textbooks and other teaching aids. As has already been argued, our current abject dependence for these on the fadism of publishers is most unfortunate. Such materials must be appropriate to the curriculum and to the competence of the body of teachers which varies markedly across Canada but should gradually improve. If university mathematicians become actively involved in the training of mathematics teachers, they will undoubtedly produce many informal sets of notes. With the aid of elementary and secondary school teachers, some of these could be worked up into permanent form and distributed by means of off-set printing or other inexpensive means. As the mathematical competence and confidence of teachers increases, we should be able to free ourselves of current widespread lock-step tactics of committing whole educational systems to one or two series of textbooks aimed by some publisher at what is judged to be a universal level of mediocrity.

An Additional Aim

Primarily, we proposed that the primary aim of the school mathematics programme should be to raise the general level of understanding by all citizens of the meaning and role of mathematics in modern society. The program in secondary schools and community colleges has an additional important purpose - <u>to lay a sound basis for the technical mathematical training of those students who will - or who are likely to - need mathematics for professional purposes</u>. In accomplishing and reconciling these aims, it is necessary that the mathematics curriculum be under continuing review. Input to the Mathematics Study from Briefs, the Seminars and the Questionnaire highlights three areas of current concern. (a) The apparent lack of relation of the current mathematics programs to applications in society or in other school subjects. It would be generally beneficial in many secondary schools and community colleges if the sequence of topics in mathematics were closely integrated with courses in Shop Mechanics, Physics or Chemistry depending on the circumstance of the school and the student. (b) Increasingly the computer is influencing all our lives. A critical evaluation of what can and cannot be done with computers might well find a place in the high school curriculum. However, the tendency in some school systems to blow this concern up into a two-year course which leaves the student with a fixation on FORTRAN, that may well mar him irretrievably for life, is going too far in our view. (c) No literate person in our society can fail to read about the statistical evidence that smoking causes cancer or that lower maximum highway speed decreases the number of fatal accidents. Nor can we avoid being bombarded by "scientific statistical" evidence that GLOWSHEEM toothpaste is 71% more efficient than any other. Certainly, Statistical Theory is one of the most widely applicable parts of mathematics and some introduction to it is appropriate in secondary schools. Those statistical ideas which would be helpful for all citizens should gradually be suffused throughout the school program from the earliest grades. The comprehensive brief[13] submitted to the Study by the Statistical Science Association of Canada has much comment pertinent to this point. A project of great interest is currently being undertaken under the direction of Professor Ian MacNeill of the University of Western Ontario to produce text materials for an introductory high school statistics course.

Recommendation

The following recommendation attempts to sum up all the preceding. We do not pretend that our specific suggestions for positive action are at all sufficient. They are all necessary but do not exhaust the possibilities that will emerge if able creative minds set to work energetically to improve mathematics teaching with the strong and persistent backing of the several governments.

The provincial Ministers of Education should take vigorous action to improve the teaching of mathematics in the elementary and secondary schools.

Notes - Chapter IV

1. The quotation is the royal command, promulgated on March 5, 1887, that initiated modern mathematical education in China. It is cited on the frontispiece of the book described in the following note.

2. Excerpts from the conclusion pages 306-7 of Mathematics Education in China: Its Growth and Development, by Frank Swetz, MIT Press, 1974. This fascinating book came to hand too late to affect the writing of this Report. Like most cross-cultural studies, it is very valuable in enabling us to see ourselves by contrast with a radically different society.

3. Torsten Husen, International Study of Achievement in Mathematics, Almquist and Wiksell (Stockholm) and John Wiley, (New York), 1967. The quoted Standard Deviations seem absurdly high but are those given by Husen.

4. Morris Kline had a distinguished career in applied mathematics at New York University where he directed research on the mathematical behaviour of radio waves. Between 1942 and 1945 he designed radio meteorology equipment for the U.S. Signal Corps and holds the basic patent for the system employed by the U.S. Armed Forces. He is the author of the lively and interesting book Mathematics in Western Culture, Oxford University Press, 1953. He is a master of polemic which he has directed against his conception of the "New Math". Unfortunately, he seems sometimes to get carried away by his own rhetoric. Such was the view of several rather conservative teachers who heard him in Toronto several years ago. However, he should be taken seriously since he has an unusually extensive knowledge of classical mathematics, a passionate concern for teaching and a vivid witty style.

5. David Wheeler is a British mathematical educator now living in the U.S.A. We quote from an article which he wrote for the Notes of the Canadian Mathematical Congress, October 1974.

6. George Boole (1815-1864) was a self-educated mathematician of great power. Bertrand Russell said of him: "Pure mathematics was discovered by Boole in a work which he called The Laws of Thought (1854)". Boolean Algebra, named after him, is an algebra of sets which now is applied in designing complex switching circuits.

7. A first draft of this paragraph was criticized for its "nineteenth century psychology". In fact, it echoes recent studies of the evolution of the brain which purport to show that the human brain has distinct parts recognizable chemically and histologically. The part which controls basic physiological process is common to the reptiles; overlaying that is the limbic system which plays a key role in our sexual and emotional responses; and, emerging late in the evolution of the human species are the large frontal lobes which enable man to rationalize. During a visit to the Bedford Oceanographic Research Institute we learned that the brain of a whale is much larger than that of a human. One theory claims that the whale needs a very large mental computer to analyze the sonar signals which he uses to explore his environment.

8. Reported to the Study by the Chairman of the Mathematics Department of the University of Winnipeg.

9. The quotation is from a chapter by Beeby entitled "Educational Aims and Content of Instruction", in Essays on World Education edited by G.Z. Bereday, Oxford University Press, 1969.

10. As part of an Ontario Remedial Mathematics Project supported by the Ontario Universities Program for Instructional Development. An interim report is available from Professor A.J. Gold, Department of Mathematics, University of Windsor, Windsor, Ontario.

11. H.B. Griffiths and A.G. Howson, Mathematics Society and Curricula, page 62, Cambridge University Press, London, 1974.

12. Under the name PERMAMA the University of Quebec has developed an ambitious program of courses by TV for school teachers. A fascinating account of this and related developments in mathematics education in Quebec can be found in Fasicule II: Parts 1 and 2, of Apprentissage de la mathematique et education prepared by Professor Claude Gaulin, of Laval, in connection with PERMAMA XVII issued by the Tele-universite of the University of Quebec.

13. For information about this, consult Professor Ian MacNeill, Department of Mathematics, University of Western Ontario, London, Ontario.

Chapter V

Teaching Mathematics in Community Colleges

> *"I have read the mathematics till I am grown perfectly stupid, and have algebraically worked away the little portion of understanding that was allowed me. They have not even left me the qualities of a coxcomb; for I can neither laugh nor sing, nor talk an hour upon nothing. The latter of these is a sensible loss, for it excludes a gentleman from all good company and makes him entirely unfit for the conversation of the polite world."* -
>
> General Wolfe[1]

According to the mathematics teachers in community colleges who wrote to the Study, James Wolfe was not alone in having difficulty with mathematics. Many students come to community colleges with an inadequate control of even the most basic elements of arithmetic, algebra and geometry. This constitutes a formidable handicap for them.

The New Factor

The sudden rise of the community colleges about ten years ago in various parts of Canada represents a most significant and welcome development in the Canadian educational scene. Hitherto the possibilities for education beyond secondary school were extremely limited apart from university. We had nothing comparable to the diversified system of technical and other specialized colleges in the United Kingdom and Germany, or the variety of community colleges[2] in the U.S.A. As a result, we have placed an exaggerated value upon a university degree, and have demanded that the universities provide types of training which they are not properly equipped to give. Hence there has been an unhealthy explosion in the number of students seeking admission to university and a dearth of young people properly trained in a wide variety of technical skills essential to modern society.

We are using "Community College" as the generic term for non-university post-secondary educational institutions such as the CEGEP's[3] in Quebec and the CAATs[4] in Ontario. Their fundamental raison d'etre is to educate students in a wide variety of skills needed by our society, not only in mechanical and electronic trades but also in various service and aesthetic sectors of the work force which are becoming of increasing relative importance.

According to one correspondent [5] in Ontario a CAAT is normally divided according to the following divisional disciplines:

1. Business
2. Applied Arts
3. Health Sciences
4. Manpower Training (Including Trades Training)
5. Technology

Within the category of Manpower Training are such instructional programs as barbering, carpentry, electrician, meat cutting, plumbing, and welding - categories that may be described as trades before the advent of Community Colleges in the mid 1960's. Some of these courses are of one year duration. Within the confines of Manpower Training is also found supporting instruction for Industry-based Apprenticeship Programs. Students, mostly with grade 12 education, (although grade 8 is a minimum requirement in some instances) are employed by companies as apprentices where part of their instruction is acquired through a specific College Program.

On the other hand, the programs that fall within the scope of Technology are divided into two distinct levels of instruction, the Engineering Technician which is a two-year program and the Engineering Technologist which is a three-year program both with specific disciplines such as:

1. Aeronautical
2. Chemical
3. Civil
4. Electrical
5. Electronics
6. Geological
7. Industrial Instrumentation
8. Mechanical
9. Metallurgical
10. Mining

In a number of the College Programs, the essential difference between a technician and a technologist is his mathematical ability to solve problems and design practical solutions in support of engineering functions under the direction of qualified engineers and scientists.

Both the Manpower Training and the Technology Divisions may be described as "Technical Education and Training" but with separate and distinct objectives requiring different levels of academic prerequisites.

In particular, College Technology Programs are closely related to University Engineering Programs where a thorough understanding and ability to use mathematical principles are essential. To acquire this knowledge, students should follow an academic stream.

Not so many years ago, industry relied quite heavily on secondary schools to produce a graduate who could adapt very quickly to a machine shop, autobody shop, or an electrical shop. There were essentially no schools between secondary school and university. As a result, some aeronautical engineers were employed essentially as draftsmen, junior electric engineers wired up prototypes and so forth. The advancements made in technologically-oriented fields today have surpassed the capabilities of most secondary school graduates and now industry is turning to the college graduate with increased knowledge and training to perform similar tasks, but with greater accuracy, more precision, and greater economy, and employing in many cases, more sophisticated equipment.

A survey of the faculty of Cambrian College revealed that many students who enter the technical programs of the College have difficulty with arithmetic, notably the handling of fractions, and too frequently they cannot manipulate simple algebraic expressions. They also noted that similar subject headings used in various secondary schools have very little connection with the subject matter actually taught and that standards of accomplishment vary widely among schools. This causes almost insuperable difficulties for the instructors in the CAAT and is a source of much frustration for the students.

In Quebec, the CEGEP's not only provide instruction in areas covered by the CAAT's in Ontario, but also offer extensive academic programs which, in mathematics, are similar to the Ist and IInd Year offerings in anglophone universities. Naturally, no one college can provide in-depth training for the full spectrum of the needs of our technological society. Some of them have notable excellence in a particular topic, e.g. the College in Burnaby, B.C. specializes in Forestry; Algonquin College in Ottawa is very good in Electronics.

Another correspondent [6] is critical of the lack of effective interaction between the CAAT's in Ontario and the universities. He believes that "the Colleges should be related to neighbouring universities which should have representatives on their Boards and plan coordinated programs. In Europe, Technical Colleges are long established and prestigious. Ours even hesitate to take a PhD on staff! This is the tragedy of our position. We have learned too little from experience elsewhere. We take over the wild enthusiasm for a university degree from the U.S.A. Even there, MIT and Rensselaer Polytechnic are old institutions with recognized standing of a high level. The tiresome dichotomy between teaching and research could sort itself out given proper standing of our community colleges. This must come, since the high schools are even more terrified of PhD's! ... Mathematics is a good subject on which to base such general criticisms of the system since it is involved in so many aspects of education, not just engineering but statistics, the social sciences and science generally."

The Corps of Teachers

Professor Robinson's suggestion that the Community Colleges and the high schools should employ more PhD's will be met with dismay in some quarters. The sort of attitudes which the traditional preparation encourages in mathematics PhD's probably means that few of them would be truly satisfactory in a College or high school. However, we are reminded of a passage in the remarkable book[7] that Wittenberg was writing in 1965:

> "How many of the teachers of the various academic subjects in Ontario high schools hold the various types of academic degrees: doctorates, master's, first-class honours bachelors, etc. ... We do not know. ... No such statistics are available. ... For all we know, it is quite possible that there may be high schools without a single really qualified specialist teacher in an academic subject. I need not point out that, if this were the case, it would be a crushing discrimination against the students in such schools. ... The graduates of our high schools will have to compete with those of schools in other countries later on - if not directly, certainly indirectly, through economic and social achievement; they will have to compete, for instance, with the graduates of the very ordinary Swiss high schools in which I taught ten years ago, in which every teacher of mathematics had a PhD, with one exception - a man who had a first- class master's degree."

Alexander Wittenberg was a profound commentator on the educational scene in North America. On every possible occasion he emphasized his conviction that the strength of any educational system is determined completely by the quality of the corps of teachers. Because of the manner in which Community Colleges were created overnight by fiat of the provincial governments, there has not been time to train a truly adequate teaching staff or to articulate a proper rationale for the functioning of the highly diverse elements which cohabit under the broad Community Colleges umbrella. We may hope that through a gradual evolution the system will steadily improve.

Mathematics in the College

If the experience of one dedicated and innovative mathematics instructor in a Community College is typical, a long road ahead must be traversed before mathematics comes to play its proper role in the Community Colleges. He has the feeling that the majority of his colleagues in the mathematics department are continually on the defensive, afraid of new ideas and largely unqualified to explore new paths. At the same time he can perceive obvious possibilities for making mathematics more relevant to the training of the students in his college.

At present in most of the Community Colleges, the function of mathematics is viewed as basically practical, e.g. accurate control of arithmetic, perhaps aided by pocket calculators, is needed for shop-work; the properties of periodic functions, especially the sine and cosine, are needed for understanding of electronic equipment; the logic of algorithms is fundamental to programming and computing.

The inadequacy of the elementary and secondary schools in ensuring that their graduates have an accurate and secure control of basic mathematics, coupled with the absolute necessity of this for many of the programs in the community Colleges has forced the latter to create new approaches to remedial mathematical instruction. The mathematics instructors in the Community Colleges tend to be more "hard-headed" than those in high schools or universities. This is good.

Fanshawe College in London, Ontario has, largely through the initiative of R. Zimmer, developed a Mathematics Learning Centre which has broken new ground for Canada. Students come to the Centre on a purely voluntary basis when they encounter difficulty with mathematics. The Centre has available a wide battery of diagnostic tests designed to pin-point the student's difficulty. If and when this is accomplished, the student's attention is directed to a variety of self-study materials in programmed-learning books or on tape. Individual guidance and encouragement is provided to enable the student to overcome his difficulties and to test his own progress. In its first year beginning in 1974, the Centre helped approximately one thousand students.

However, there is no inherent reason for the mathematics curriculum in a Community College to have a purely utilitarian aim. This may be necessary and sufficient for students whose sole interest in mathematics is to use it as a tool in connection with some technical pursuits. However, the Community Colleges are increasingly serving as centres for the continuing education of persons intensely concerned with broadening their cultural and intellectual horizons. At least one Community College in Vancouver offered a wide-ranging course in the history and social significance of mathematics which was extremely well-received.

Unfortunately the Study Leaders were unable to devote the time and energy necessary to gain a real grasp of the actual condition of mathematics and its teaching in the Community Colleges. Nor did we receive sufficient information about mathematics in Community Colleges, through briefs or other means, to be in a position to make meaningful recommendations on this topic.

However, we are convinced that the Community Colleges occupy a most strategic position for the effective deployment of mathematics in the service of Canadian society, and we urge that if a Council for the Mathematical Sciences is brought into being as a result of this Study, then it should give very high priority to instituting a thorough subsidiary study of mathematics in the Community Colleges.

Notes - Chapter V

1. This is from a letter of General James Wolfe to his father taken from Wolfe of Quebec, White Lion Publishers, London and New York, 1973, p. 95. Note that "sensible" is used in its former meaning of "significant".

2. The first National Convention of Two-Year College Mathematics Departments in the U.S.A. was held in New York City in April, 1974. It was attended by 289 representatives of 120 colleges in 28 states. The MATYC Journal - the organ of Mathematics Associations of Two-Year Colleges - is in its ninth volume. Its stated purpose is to "facilitate the exchange of ideas pertinent to mathematics education at the two-year college level".

3. College d'Enseignement General et Professionel.

4. College of Applied Arts and Technology.

5. G.C. Field, P. Eng., Chairman of the Electronics Instrumentation Department of Cambrian College of Applied Arts and Technology in Sudbury.

6. Dr. Gilbert de Beauregard Robinson is an emeritus professor of the University of Toronto. He is widely recognized as the world authority on the symmetric group and its representations. He concluded his distinguished career at the University of Toronto as Vice-President for Research Administration.

7. The Prime Imperatives: Priorities in Education, p. 34 (Clarke, Irwin and Company Limited, Toronto, Vancouver, 1968) is a probing analysis of education in Canada by Dr. Alexander I. Wittenberg, the internationally known authority on the philosophy of science and education who studied and taught in Switzerland. He joined the faculty of Laval in 1955, later moved to York University and was actively involved in improving mathematics education in Canada until his untimely death in 1965.

Chapter VI

Teaching Mathematics in University

"The primary reason for the existence of universities is not to be found either in the mere knowledge conveyed to the students or in the mere opportunities for research afforded to the members of the faculty.

"The justification for a university is that it preserves the connection between knowledge and the zest of life, by uniting the young and the old in the imaginative consideration of learning. The university imparts information, but it imparts it imaginatively. At least, this is the function which it should perform for society. A university which fails in this respect has no reason for existence." - A.N. Whitehead[1]

In the present chapter we discuss the teaching of mathematics in Canadian universities. Clearly the success of the schools and Community Colleges in communicating mathematics depends crucially on the knowledge and attitudes which the university imparts to those of its students who become mathematics teachers. However, among the respondents to the Questionnaire, 60% of the Bachelors, 43% of the Master's and 12% of the Doctors are working for industry or government, so it would be unwise for the program of university mathematics departments to be geared, explicitly or implicitly, for the training of mathematics teachers only.

However, whatever we do in the university, if we are not to fall under Whitehead's condemnation, our students should emerge with imagination kindled and with zest to explore the beauty, practical utililty and wide-ranging outreach of mathematics.

Undergraduate Programs

Comments on undergraduate mathematics came to the Study from three sources: (a) The Seminars, (b) Questionnaire, (c) Briefs from a variety of persons - principally university professors. Of course, there was not an absolute unanimity of views. However, there was a majority opinion which surprised us by the conviction with which it was held by quite diverse groups in the population sampled by the three techniques enumerated above. It is our impression that 90% of the participants in the Seminars from outside the university and, depending on the issue, 65% to 85% of the respondents to the Questionnaire held the following opinions rather firmly.

(a) University programs should not be regarded as technical training schemes to produce a neatly embossed product to fit perfectly into a predetermined slot. In particular, the most valuable effect of studying mathematics is seen to be the inculcation of a way of thinking, and a power of analysis which seldom results from other forms of training. There was widespread acknowledgement and appreciation of the fact that the mathematics departments in Canadian universities have been quite successful in enabling many of their graduates to gain these invaluable abilities.

(b) However, most of these graduates are unable or are afraid to take initiative in unfamiliar situations. If a well-defined mathematical problem is proposed, which falls squarely into a category with which they are familiar they can and will gladly solve it. But they have little ability to tackle unfamiliar, or to model ill- defined real life problems. Their training has consisted too much of absorption and regurgitation. It has not developed in them confidence in their own ability.

(c) Furthermore, our graduates are generally sadly lacking in the ability to communicate and work with others. This is due partly to their poor command of English, commonly, to their unfamiliarity with any science other than mathematics, and to the almost exclusively individualistic nature of the study process as it is practised in Canadian high schools and universities.

(d) Apart from basic linear algebra, statistics and computing and the concepts of derivative and integral, it is hard to argue that any other element of the normal honours mathematics program - e.g. complex function theory, Galois theory of fields, Euler's equations - is essential for students who will later use mathematics in their jobs. However, it is desirable that they develop the ability to "pick-up" a new mathematical technique or theory if and when they need it and not be intimidated by such a necessity.

It will not be easy for mathematics departments to modify their programs so as to effectively meet all the desiderata suggested by the above comments. Some of the factors which militate against this are deep-seated in Academia and not peculiar only to mathematicians. We attempt to explicate only the more obvious.

Even though the infamous slogan "publish or perish" has been much less applicable to Canadian universities than to universities in the U.S.A. with outstanding or aspiring Graduate Schools, it points to an attitude which dominates the spirit of most academics. Their highest loyalty is to their discipline. They will gain prestige and the respect of their peers by research in their discipline. This means adding some new fact or theory to the discipline. A good professor is an enthusiast for his discipline. If he were not and had any self-respect and ability, he would change disciplines. Hence, the university often appears not so much as a community of scholars, but as a collection of Departments pursuing fame in their several disciplines. In this context, the ultimate object of teaching is to lead the student to the highest level in the discipline of which he is capable since, it is argued, this is the best education. Only by intense application to a well-defined subject matter will the student get beyond the superficial level of jack-of-all-trades - master-of-none.

Although there is great force in this last argument, it is often used inappropriately. Too frequently the basic guiding principle, implicit in many structures of Honours programmes, has been that the supreme good is a replication of the species <u>mathematicus academicus</u>. The effect of this is to direct the main interest and concern of the department to the teaching of those few who conceivably are destined to the high calling of professors or research mathematicians. All others are really of no importance and are expected to be content to feed on the crumbs left over from the superb banquet on which only the select could hope to satisfy themselves directly.

The attitude which is being parodied here is especially absurd when it appears, as it does in nearly every Canadian university, among mathematicians, but it is endemic to nearly all Departments in Arts and Science and is at the root of much of the malaise affecting the university in the Western world. It would be foolish to count on the immediate disappearance of so all-pervasive an attitude. However, we may hope that Canadian mathematicians will succeed in approaching this issue more rationally than some of their other colleagues.

Another factor which tends to result in an overemphasis of factual content, rather than attitudes and ability to mathematize, is that a good mathematician loves mathematics. He feeds on mathematics with a greater lust than the lotus-eaters feed on lotus or the wine connoisseur enjoys a Corton Clos du Roi, 1961. He cannot understand that others may not obtain the same satisfaction he does. This is especially characteristic of young instructors. They are often "carried away" with their subject. From a human point-of-view, many students are delighted to witness such enthusiasm. One is more encouraged about the human race by watching an enthusiast for something as relatively harmless as mathematics than by listening to a bored cynic! Even when they hardly understand anything, students will speak warmly about their instructors' enthusiasm. They may learn little mathematics, but they are encouraged to believe life is worthwhile. This is not to be sneered at!

Of course, it is much easier to teach mathematical facts and theories than how to communicate or how to work together. In order to avoid these latter more difficult tasks, the mathematics professor will say that these are the responsibility of the high school, of the English Department, of psychologists, of the Boy Scouts, of the home and of the Church. In any event, not the responsibility of the mathematics department! After all, the mathematics department, as everyone knows is responsible solely for the Discipline! Our task is to teach the meaning of group, field, real numbers, filters, statistics, hilbert space, and differential equations, and occasionally their applications. We know from experience that even this task is almost impossible. Clearly, no one else is competent to attempt it. It is unreasonable to expect us to do more ... so, the argument often goes when we mathematicians gather together.

Further, it is much easier to examine students on whether or not they can solve a well-posed mathematical problem or prove the Heine-Borel theorem, than to examine them on their ability to formulate models of ill-defined situations, to communicate their ideas or to work with others. If we were to venture into these uncharted regions, our marks would lose their present pristine objectivity. We would have to make subjective judgements of the vague and unsatisfactory type for which we are constantly criticizing our colleagues in the humanities and social sciences.

There are good and bad reasons tending to prevent mathematics departments from interpreting their task as anything other than providing a technical mathematics training. However, the signal received by the Mathematics Study from many diverse sources is ambiguous. "This you should have done, and not left the other undone."

What can we do?

The briefs and the Seminars suggested many specific ideas. However, each particular mathematics department must find its own way, given the insight and abilities of the members of the staff, the aims and quality of the students and the resources which are available locally. There are few - if any - universities in Canada which would not profit from a critical examination of their mathematics program in the light of the comments which have come to us through the Study and are summarized above or suggested by the results of the Questionnaire reported in Appendix I. We venture the following suggestions:

1) Almost all mathematics professors, when pressed, allege that their highest ambition in undergraduate teaching is to convey not specific content but rather a way of thinking. It was this way of thinking which we previously referred to as mathematizing. However, much of our effort is directed at lecturing students and then testing them not on their ability to mathematize, but on their skill in regurgitating the content of the lectures. This habit of university staff derives from the medieval practice at the University of Bologna where the students contracted with the lecturer at the beginning of the term as to exactly how many pages of the prescribed text he would read to them (only the professor had a copy). If at the end of term he had not fulfilled the contract, the students did not pay their fees!

We need to devote more time and effort explicitly to the task of encouraging - indeed, forcing - students to mathematize. In doing this, it will certainly be necessary for them to learn specific pieces of mathematical lore. The mathematical content of the course would be chosen as a result of experimentation to discover the most suitable material with which the students could develop their mathematical powers. There is a slogan which is quite apposite in this context - higher intellectual skills can be learned but not taught. Learning to think mathematically is not easy for the average student - many of whom want the professor to write everything on the board to be copied, protest that it is unfair to examine on any topic which was not treated fully in lectures and study with the purpose of dishing back on examinations more or less verbatim the material "taught". Exposition-regurgitation is the method which demands the least creative effort from staff and students and is therefore an irresistible universal temptation.

However, for mathematicians, there is a special temptation. A topic such as the theory of Complex Functions, Galois Theory of

Fields, Convex Programming or C* -algebras can have extraordinary elegance and the beauty of a Bach fugue. There is great joy for the lecturer in expounding and for the student in witnessing the development of such a topic. But contemplation of the beauty of great mathematics is not the active demanding work of mathematizing. Indeed, contemplation can cut the nerve of action. There was a critical moment in the early history of the Society of Jesus when Ignatius Loyola peremptorily ordered the Spanish monks to stop praying so much and minister to the sick and the poor. For many students, the evil habit begins in high school where they study only complete mathematical theories and gain the conviction that all mathematics is polished and neat.

The practical point which is being made here has been made many times before. Mathematicians should put much less stress than they are wont to do on expounding large, logically coherent portions of mathematics. Rather, they should employ techniques of teaching which force students to think for themselves. Hence more emphasis should be placed on:

a) the origin and background of problems;

b) the intuitive, trial-and-error methods which all mathematicians employ to actually solve problems, rather than swift exposition of a neat solution;

c) problem-solving by students in groups and in tutorial sessions;

d) independent study projects by individuals or groups of students.

Certainly this path is not as easy for the professor as the classic exposition-regurgitation process. It will almost certainly meet resistance from some students. However, if reasonable examining methods are employed and if the value of the skills which will result are explained carefully, the enthusiastic participation of students can be gained. And, in the final analysis, successful doing is more rewarding than mere contemplation - to mathematize is to joy!

2) We mathematicians must relate more effectively to other disciplines - physics, chemistry, engineering, economics, psychology and biology. Each mathematician should strive to have an informed understanding of at least one other such discipline at a level which will enable him to discuss research papers involving the application of

mathematics with university colleagues in the other discipline. In this way, an average sized department will be able to relate meaningfully to all the users of mathematics in the university and be able to convince students of the relevance of mathematics to their principal discipline (if it is relevant!).

To the inevitable cry that the young mathematician will not have time to learn another discipline and also write papers in his own subfield of mathematics, there are two responses. (i) He should have learned something else besides mathematics during his undergraduate and graduate training - a point to which we shall return. (ii) Developing such outreach into other disciplines is much more important to the university and to the mathematical community than many of the papers written by mathematicians in Canada or any other country. Further, most mathematicians will derive greater satisfaction from real interdisciplinary interaction than from publishing a paper which is read only by themselves and the referees.

3) Honours mathematics students should be required to pursue at least one other subject in which mathematics is employed in a significant fashion. Their study of the subsidiary subject should be monitored by the mathematics department to ensure that they gain an understanding of how mathematics is relevant to it. It has been argued that there is occasionally a student who is only good at and/or interested in mathematics and if he wants to be a pure mathematician, he should not be forced to study anything else. However, in the foreseeable future, unless he is a Gauss, a Hilbert or a von Neumann, he is unlikely to get a job in Canada unless he can talk to someone other than a mathematician. If a future Gauss did turn up, he should be recognizable and an exception could be made. Of course, Gauss, Hilbert and von Neumann were all quite competent in disciplines other than mathematics!

4) Most English and French Departments do not undertake the dull and difficult task of teaching language as a tool for communication. This would be hard and uninteresting work so they prefer to treat life and its values as seen through literature. Therefore, if graduates of mathematics departments are to be able to communicate with the rest of the world, the mathematicians will have to teach them how to do so since in recent years the elementary and secondary schools have been less than successful in carrying out this task. Three practical methods: (a) Written exercises should be marked stringently for format, for grammatical form and spelling. (b) Each year all students should submit a minimum of two short essays on the history or applications of mathematics. (c) Approximately half of

the exercises should be submitted by groups of three to five students who will be urged to meet to discuss the solutions, explaining them to one another. In rotation, one member of the group would be responsible for writing up and submitting the solution.

5) For most mathematics departments, the guiding principle in shaping the entire spectrum of undergraduate offerings has been to create a set of courses for the Honours stream. This is usually a sequence of at least eight and, allowing options, often as many as twenty courses. At Queen's, indeed, there are twenty-four courses offered primarily to Honours students. In this respect, Queen's is merely typical of the larger departments such as Dalhousie, Laval, McGill, Toronto, Waterloo, Manitoba, Alberta and UBC. This preoccupation with honours courses is such a universal mind- set that even some of the small newer departments devote 50 percent or more of their teaching effort to the courses of the Honours stream - even though they have only 2 or 3 math majors in third and fourth year while they have the opportunity to teach several hundred users of mathematics.

How can this absurd situation be explained? Very simply: the herd instinct! Between 1930 and 1960, the Mathematics, Physics and Chemistry Course at the University of Toronto was a magnificent program which attracted a very large proportion of the ablest students of Ontario (winners of scholarships in History or Classics who came to University College frequently ended up in M., P. and C.). It gained a high and well-deserved reputation by an extraordinary performance in the Putnam Competition and by the success of its graduates in the leading American graduate schools. The rest of the mathematics departments aped Toronto. They were frequently staffed by its graduates.

Furthermore, every young PhD has an understandable desire to teach an advanced course in his particular topic. Before 1950, there were very few graduate students in mathematics in Canada so everyone wanted to teach a senior honours course. Because the main loyalty of every "good" academic is to his discipline, we were convinced that it was - and is - our duty to expound every possible aspect of mathematics for all those - rather few - who had the perspicacity to listen to us. At the same time, we were satisfying our procreative instinct - replicating the species. (Indeed, is it not a Biblical injunction "be fruitful and multiply"!)

In such a context, the sobriquet "service course" connotes a tedious imposition which diverts the mathematician from his true calling. This, despite the fact that it is the students in "service" courses who will make effective use of mathematics for the good of society, who justify our salaries and from among whom many of the leaders of government, industry and finance will be drawn. The undergraduate experience of these students with mathematics will be the greatest single factor in shaping the future attitude of society to the mathematical community.

The practical consequence of the preceding considerations is that most Canadian mathematics departments should revamp their course offerings in order to provide the best possible set of courses for the users of mathematics. When this has been accomplished, thought can be given to a supplementary program for the training of intending pure and applied mathematicians. Such a program for Honours students should emphasize not lecture courses but rather independent study, which encourages the students to learn to read and synthesize mathematics and tackle problems on their own or together with fellow students.

6) In order to break the current isolation of academic mathematicians from those who use mathematics as a tool, we must create many new channels of communication between the mathematicians in universities and community colleges on the one hand and the users of mathematics in business, industry and government on the other. At the student level, the placement of students in mathematically significant jobs during the summer, or during the year by such schemes as the Cooperative Programme of the University of Waterloo, is an important step in the right direction. However, if curriculum is to be relevant and courses are to be taught properly, then university professors need first-hand experience in the situations in which mathematics is deployed. To this end, it must become much easier for people to move back-and- forth for short periods between university, community college, government, business and industry. It is hypocritical for people in industry and government to accuse professors of being in an ivory- tower if they erect impenetrable psychological and practical barriers to effective interaction. Assuming that, on the average, 15 percent of a professor's annual salary would cover travel and other out-of-pocket expenses needed for a displacement of two to six months, one can estimate that if 3 percent of the salary budget were available to expedite such ventures, then each staff member could have such an experience every five years. If this expense were shared among universities, industry and government, it would be small in comparison with the long-range benefits which all parties could expect. However, the major immediate impediment to any such scheme is the belief of young

faculty members that the only item of real significance in determining their prestige and promotion is the number of papers they publish. Therefore, if a scheme of the above nature is to get off-the-ground, the reward system in the university needs to be dramatically revamped. This point will be discussed in greater detail in Chapter VIII.

To get the ball rolling, we need more universities and industries where conditions are propitious to take the initiative. Further, at the federal level, it should be possible for MOSST to cut through the red tape of the regulations of the Public Service Commission and immediately arrange for a number of departments of the federal government to experiment with exchange arrangements with mathematics departments. Initially, a variety of approaches should be attempted as pilot-projects in order to develop a set of regulations which will effectively promote real interaction while satisfying all necessary factors of public accountability.

The PhD Crisis

In 1961, Canadian universities awarded eleven PhD degrees in mathematics and in 1973, ninety-four. In 1960/61 the NRC gave $87,500 for the support of mathematics research and in 1972/73, $2,461,500. These numbers indicate that there was an astonishing explosion of graduate study and research in mathematics during those years. A considerable portion of the NRC funds were used for partial support of graduate students. In 1961, there were about 250 mathematicians in the universities at the rank of Assistant Professor or higher. By 1973, this number had increased to approximately 1,300.

From Appendix I, we see that of the 168 Canadian PhD's who responded to the Questionnaire, 82 percent are employed at universities or colleges. This is consistent with the general impression in the mathematics community that the majority of PhD candidates in mathematics are hoping to find academic positions. In the past, most of them have done so. However, the recent austerity in university financing has brought a sudden halt to the expansion of mathematics departments. Further, the rapid increase in the size of the departments during the 1960's took place mostly at the lower ranks and younger ages; so that in the next decade relatively few faculty members will be retiring.

These factors conspire to create the PhD crisis which has shaken the academic mathematical community in the past three years. According to Appendix III, between 1960 and 1970 the number of Assistant, Associate and Full Professors in Departments of the Mathematical Sciences increased by over 900. In the same period, Canadian mathematics departments produced about 360 PhD's - a deficit of over 500, which was partly filled by Canadians who studied at U.S. graduate schools. The corresponding figures for the years 1971 to 1973 are as follows:

	1971	1972	1973
Increase in Professors	80	38	27
Number of Canadian PhD's	86	88	94

The numbers cited above force one to recognize that the difficulty of lead-time makes the problem of formulating a rational viable manpower strategy almost impossible. During the 60's, nationalists were calling for Canadian self-sufficiency in producing university faculty. The 1965 Canadian need for mathematical PhD's was finally met in 1971, precisely at the moment when it became apparent that we would need less than 50 percent of the current production for the foreseeable future! An aphorism of Malcolm Muggeridge about the Dean and Chapter of Westminister Abbey can, perhaps not inappropriately, be slightly varied to read "What the Provincial Governments approve today, history will have abolished yesterday".

The academic mathematicians are not alone in being caught off-balance by adopting value schemes and developing practices appropriate to a regime of exponential growth which has suddenly been turned off. The figures quoted for 1971, 1972 and 1973 suggest that approximately half [2] of the Canadian mathematical PhD's for those years have not found regular academic positions in Canada and they have little prospect of doing so. This fact causes them considerable psychological suffering since they will not be able to fulfill their hopes. It has also sent a shock-wave through the mathematics community forcing us to re-evaluate the PhD process. Such a re-evaluation has been long overdue, but mathematicians are only human and it seems to be part of human nature to refuse to see the writing-on-the-wall before calamity actually strikes.

The calamity which struck Canadian mathematicians [3] in 1971 or 1972 arrived in the U.S. a couple of years earlier and hit the North American physics community six or seven years ago - with the retrenchment on NASA projects. At the Annual Meeting of the AMS and of the American Mathematical Monthly there has been much excellent discussion of the proper inferences we should draw from the recent course of events.

The Nature of the PhD

The nature of the PhD process has varied in detail from country to country and from epoch to epoch. There is also a considerable difference in what is expected of candidates in different disciplines. Let us think only of mathematical sciences.

On the one hand, the PhD degree is supposed to indicate that the successful candidate is a creative mathematician who in his thesis has made, and in his subsequent activity will be capable of making, original contributions to mathematics - a potential Gauss! On the other hand, it is the union card for entrance into academia. These are the two universal traditional aspects of the PhD. Every respectable mathematics department insists that an acceptable thesis "must be original and be of such value as to merit publication". Every respectable university expects that regular faculty members have a doctorate.

These two absolute conditions are now strongly at odds. Possibly, they were reasonable at Goettingen in the nineteenth century, when there was only one professor, Gauss, Riemann, Klein, and when only an infinitesimal elite attended university. Perhaps they made sense before 1900 when it was still possible for one mind to range over the whole of mathematics and encompass most of physics and several other subjects.

However, in the past four decades with the rise of mass education, concomitant with the fissiparation of mathematics into minute sub-areas, the traditional view of the PhD has increasingly revealed itself as completely inadequate. The distinguished Chicago algebraist and former Canadian, I.N. Herstein [4] is only one of many mathematicians who have called for a reappraisal of the PhD process. The current employment crisis points up very vividly one of the most blatant evils in our training of PhD's. Many of them gain a competence which is so narrow and specialized that they have very little flexibility or adaptability. They unconsciously pick up the belief that the only worthwhile professional objective is a relentless pursuit for the remainder of their life of the particular sub-sub-topic of mathematics with which their thesis dealt. Since the number of truly original minds seems to be very small, in order to satisfy the absolute desideratum of "originality" the only sure method with an "average" PhD candidate is to lead him into some hidden pathway on which no one other than his supervisor has ever trodden (or wished to tread!). There, almost inevitably or with some assistance by his supervisor, he will stumble on a charming violet which no one else has ever noticed. Perhaps the search takes two or three years; years of great emotional tension since many students feel the exercise is essentially pointless; years which could much more usefully be spent in gaining a large view of mathematics and its applications.

If the new Doctor becomes an Assistant Professor, the Dean decrees that he will not be promoted unless he publishes and begins to teach graduate students. But in what area can he supervise a prospective graduate student? Only in his speciality, or an even more arcane sub-area of his speciality.

A study of the class of 1950 mathematical PhD's in North America showed that about 30 percent published nothing or at most their thesis. At most 20 percent proved to be "creative" mathematicians according to any of the normal criteria. This leaves 80% in a depressing psychological limbo. It means that the PhD process is disastrously inefficient in achieving what mythically has been the universally avowed purpose determining its rituals. This has been patently clear for two or three decades. But we have rushed on like lemmings controlled by herd instincts and self-centered satisfactions.

The writing is on the wall perfectly clearly for anyone to read. The story is told by the figures quoted at the beginning of this chapter. Canada needs at most a very small number of the type of mathematical PhD's we have been producing in the past two decades.

However, we should not conclude that therefore no more mathematical PhD's are needed. There is little doubt that we need many more than we have been producing - but only if they have different training, and different aspirations. It will require an extraordinary effort of reorientation on the part of the mathematical community to move out of our traditional procedures and modes of thought. But something significant could happen within three to five years and within ten years we could change the Canadian graduate schools in basic and salutary ways, if there is a will to do so.

The history of the modern world for the past two hundred years plainly reveals that mathematics has played an increasingly crucial role in the development of our technological society, in the understanding of the economic forces which control our means of livelihood, and in analyzing the complex patterns of interrelationships between man and nature which determine the possibility of survival of the human race with satisfaction and decency. The rate at which mathematics - of increasing sophistication and abstraction - has been spreading into nearly all other intellectual disciplines has been most striking. We need not less but many more persons with the highest possible competence in mathematics who have the capacity to create and make meaningful use of mathematical models of the physical, social, economic and biomedical experiences of mankind. And we need university faculty who can train such persons thoroughly and soundly.

The Lamontagne Committee [5] and many other groups and individuals in Canada have asserted that in all scientific areas our R and D efforts have put a disproportionate weight on Research and insufficient on Development. The practice of mathematics in North America, as exemplified by the American Mathematical Society and by the criteria for promotion within the "leading" mathematical departments, seems to have placed 99 percent weight on Research and 1 percent on Development. That is, most pure and applied mathematicians have not felt it to be their duty to interpret the significance of their new results for the benefit of anyone other than their peers. In this context, "peers" means a minute invisible college of cognoscenti with whom they are in verbal communication. According to a somewhat cynical flippancy of a former editor of a reviewing journal, the papers which gain them promotion and prestige are seriously read on the average by slightly less than one person apart from themselves and the referee! It is seldom that a contemporary research mathematician tries even to interpret his results to colleagues in a different branch of mathematics and certainly not to physicists or economists. There are occasional exceptions: Hermann Weyl and John von Neumann in the past; Michael Atiyah currently. But these are exceptions which prove the rule.

The French tend to pursue logic to an extreme. So Bourbaki [6] has provided the purest example of the attitude we are decrying. To be fair, Bourbaki has done more than any other group to encourage fruitful interrelations among distinct branches of mathematics. But in the past they seemed to pride themselves, to exult even, in being so single-minded, so devoted to pure mathematical rectitude as to disdain any mention of possible applications of mathematics. No matter how reasonable a few of the Bourbakists have been in private, matter how reasonable a few of the Bourbakists have been in private, such has been their communal stance.

Nonetheless, it is the two Frenchmen, Grothendieck[7] and Chevalley[8] who now feel that mathematics is the running-dog of the military-industrial complex. Possibly in private, their old habits and their love of mathematics secretly take over, but not in their recent public attitudes. Ten years ago many of us would have named Grothendieck together with Gelfand as one of the two greatest living mathematicians. So Grothendieck's about-face has been a shock to the mathematical community. How dare we lesser mathematicians avoid facing the question "Is Grothendieck right?"

In fact, his recent attitude is not unreasonable when seen in the context of the myths about graduate study and research which have dominated academic mathematicians in recent decades.

Fortunately, most mathematicians are not as consistently logical as the French, so that our activities have not been totally dominated by the attitudes criticized above. If they had been, the ship of mathematics would have sunk thirty years ago. In fact, it has been floating along quite well and only now is it listing rather perceptibly.

The PhD Reinterpreted

If the above argument is essentially correct, then we need to embark on a reorientation of the PhD process; possibly along the following lines.

(i) We should give a broader interpretation of "originality" in the thesis. Rather than on insisting on the literal interpretation that the thesis must contain "new" mathematical results, we should require that it provide clear evidence that the candidate has some originality of mind and an imaginative approach to mathematics at a profound level. "New" mathematical results could provide such evidence but not necessarily. New ideas concerning the use of mathematics or about the interrelation of different parts of mathematics could equally well provide such evidence.

(ii) We should recognize that a practitioner in any of the mathematical sciences is a "professional" in the large sense and that the PhD is a professional qualification.

(iii) The PhD process should be reoriented to encourage the development of the type of persons of the highest mathematical competency which our society needs. It should be noted that, as is apparent from the remarks of the Cuban and Brazilian recorded in Chapter III, the needs of different societies vary so that there is no universal ideal image for the PhD.

(iv) The PhD process should include much greater emphasis than is currently normal on what we referred to previously as "development", which involves powers of exposition, communication skills and, at least, a modest command of the ideas and terminology of one actual or potential area of application of mathematics.

(v) The successful PhD should have an enlightened understanding of the history and social role of mathematics.

(vi) There should be active encouragement of the production of theses supervised by committees representing mathematics and one or more additional disciplines. This will help bring into being the new type of out-going mathematician we need and will also encourage current faculty members to understand one another and work together across the present disciplinary interfaces.

The reader will note that our proposals for revamping the PhD are very similar to those for improving the program for the honours undergraduate program. The chief difference would be to require of the PhD a much greater capacity for independence in formulating and solving problems and a more highly developed appreciation of the full sweep of mathematical thought. He would be an expert in interpreting to school children, to teachers, to managers in industry and government and to scientists in other disciplines what it means to mathematize.

<u>The Master</u>

For some years, perhaps even for one or two decades, it might be economically and psychologically easier to attain a partial accomplishment of the above goals at the Master's rather than the Doctoral level. Because of the peculiar aura which surrounds the PhD there is a widespread feeling in industry and government that the holder of the degree is "overqualified". Whenever a person is regarded as overqualified either by himself or by others, there are usually great attendant psychological difficulties.

Certainly, there are many situations in teaching, in industry and in government where a person with a Master's degree would now be appropriate even though twenty years ago the corresponding position might have been quite adequately filled by someone with a Bachelor's degree. The extra year or two of advanced study allows the student to round out a training involving substantial exposure to mathematics and one or more subsidiary subjects. Possibly some of the aims previously set out for the undergraduate program could be more surely attained at the Master's level.

Doubtless, the period of study for the Master's degree will continue to be an opportunity for students and professors to gauge whether the candidate should attempt the PhD. However, it can and does fill an important role in its own right. The Master should not be regarded as a failed PhD, as sometimes tends to be the case especially in the prestigious U.S. centres of Graduate Study. Departments which offer Master's programs should give careful thought to their aim, structure and content.

It would not be profitable for this Report to attempt to outline detailed programs for graduate study. The history of the university demonstrates that substantial curriculum changes can only be effected slowly. Further, a curriculum is worthlesss unless it is understood and approved by the professor responsible for teaching it.

Currently, it seems to us that many professors are mesmerized by a narrow interpretation of the requirement that a PhD thesis demonstrate originality and feel incompetent and insecure if faced with the task of supervising a thesis on a topic outside the confines of their narrow speciality. Yet in wartime, algebra professors quickly became experts on exterior ballistics; topologists did effective work in Operations Research. Most university mathematicians are highly intelligent. They can learn almost anything they really wish to learn. To accomplish the transformation of graduate study which we are calling for will certainly require readjustment of aims and activities on the part of many of us. Being human, we will resist, because it is easier and one feels safer to carry on in familiar ruts. But in the long run, changes along the general lines proposed in this Chapter will be to the great advantage of society and to the mathematical community. Helping to produce such changes can bring great personal satisfaction to many mathematicians of all ages.

Hopefully, we mathematicians will quickly grasp this nettle on our own initiative. But we may well be encouraged by Ministers of Education, by Presidents and Deans of universities and by the policies of federal and provincial governments for supporting research and graduate studies along the lines suggested here.

Recommendation

In order to initiate action by Canadian mathematics departments on the issues raised in Chapter VI, the Joint Committee recommends that

-on the initiative of the Canadian Mathematical Congress, the six sponsoring mathematical organizations appoint a task force to study the programmes in mathematics offered in Canada to Undergraduates, Master's and Doctoral students and to recommend the structure and content of such programmes which it deems appropriate in Canada at present and for the immediate future.

Notes - Chapter VI

1. From Chapter VII of The Aims of Education and other Essays, Williams and Norgate, London, 1932. Alfred North Whitehead had three careers: at Cambridge, where he taught Bertrand Russell and with him wrote the remarkable three-volumes of Principia Mathematica on the logical foundations of mathematics; later at Imperial College, London, when he developed a physical and philosophical theory of relativity alternative to Einstein's; and for twenty years after the age of 63, at Harvard where he articulated one of the most subtle metaphysical systems of this century which found expression in his difficult work Process and Reality. In The Aims of Education he inveighs against dead knowledge and inert ideas, insisting on the fact that students are alive and that the purpose of education is to stimulate and to guide their self- development. The book is largely a collection of lectures given over the period 1912 to 1928. It is still the most profound and stimulating discussion of mathematics education available to us.

2. Appendix I contains more detail about the Canadian PhD's of the class of 1973 and also the structure of the mathematics faculty. The total number reported for the faculty is almost certainly too low. (cf. Appendix III), however the information which it provides about the flow of persons in the Departments which replied to the Waterloo Questionnaire is quite interesting. Indeed, it would be worthwhile to obtain such information on a regular basis since the changes reported in the flow diagram are perhaps more important for Manpower policies than absolute numbers in each of the basic categories.

3. The surplus of PhD's has not been uniformly critical in all branches of the mathematical sciences. There still seems to be a dearth of Canadian PhD's in Computing Science and in Applied Statistics. It is of course important not to panic. We have just passed through a phase when many people thought that Canada was producing too many engineers but now there are informed voices predicting a crippling scarcity of some types of engineers.

4. I.N. Herstein, "On the Ph.D. In Mathematics", American Mathematics Monthly, 76, 1969, pp. 318-824.

5. The Honourable Maurice Lamontagne was Chairman of a Committee of the Senate of Canada which produced a three volume report on A Science Policy for Canada - available from Information Canada.

6. See the section on the Mathematical Sciences in Chapter II.

7. Alexander Grothendieck won the Field's Medal in 1966 for his profound contributions to algebraic geometry. A few years ago he founded a society called Survival concerned with the large problems, such as population, pollution and peace, which mankind faces. Then he decided that mathematics is chiefly a tool in the service of the power-elite and in the past three years has even attempted to dissuade young people from studying mathematics because of the "immoral" way in which their knowledge would inevitably be exploited.

8. Claude Chevalley is a distinguished algebraist and is especially noted for his three volumes on Lie groups and Lie algebras. He was one of the original members of Bourbaki.

Chapter VII

Research

"I am part of all that I have met;
Yet all experience is an arch wherethrough
Gleams that untravelled world, whose margin fades
For ever and for ever when I move."

\- Tennyson[1]

In the two preceding chapters there have been occasional negative remarks about certain attitudes towards mathematical research. But, let there be no doubt, research is the life-blood of the mathematical enterprise. A mathematics department which is not commited to active research on core mathematics cannot remain alive very long. The same insatiable craving which drove Ulysses beyond his known world has forced mathematicians to create the architectonic structure which is ours in the mathematical sciences to-day. Whitehead argues (in the locus cited in Note 3, Chapter II) that modern mathematics surpasses all the higher achievements of the human spirit, with the possible exception of music. Whitehead - the mathematician - may be biased, but all will agree that in mathematics we have a marvellous cultural treasure.

However, it also provides practical intellectual tools. For the paradox is well established that, by pursuing the utmost abstractions of mathematics, we have gained understanding and control of concrete fact. Mathematical research was the root source of many important other developments:

-radio, television, the astonishing views we have recently obtained of the surface of the moon and of Mars were possible only because of the mathematical theory of electricity and magnetism due to Laplace, Kelvin and Maxwell.

-the design of telephone exchanges and of computers employs algebraic ideas involving complicated and detailed extensions of the logical and set-theoretical work of George Boole.

-space-ships have been guided with fantastic accuracy to landing-sites on the moon with the aid of precise solutions of the differential equations which express Newton's mathematical model of the solar system.

-modern methods of detecting brain-tumours or heart disease depend on the analysis of wave-forms by methods originated by Fourier for the study of periodic functions.

Most Canadians have, at best, only the vaguest notion of the subtle but often crucial manner in which advanced mathematics undergirds their lives; and the amount and variety of such mathematics has been growing exponentially for the past hundred years. There is no sign of abatement. Mathematics steadily makes inroads into more and more areas of our life.

The primordial source of all this is research carried on by mathematicians in the past and present. Their mathematical theories gave physicists, chemists, engineers, biologists, economists, etc. intellectual tools without which they could not understand or control our environment.

What Motivates a Research Mathematician?

This question has already been touched on in Chapter II. It is not easy to answer it with absolute assurance. For one thing, motives vary markedly from one person to another. One of the best sources to help the layman understand how a mathematician works and thinks is to be found in a recent biography of Hilbert [2]. Hilbert's achievements in the period 1890 to 1920 place him among the greatest of all mathematicians, possibly second only to Gauss and certainly on a par with Weierstrass and Poincare.

From the accounts of the life and work of such men and women, one quickly learns that mathematics can be intensely beautiful; mathematics can fascinate you just as a puzzle can absorb your attention so intensely that nothing else diverts you; mathematical problems can challenge you with a force as great as that which impels the mountaineer to attempt the ascent of Mount Everest; and mathematics can be useful in answering questions thrown up by nature or society and which pique your curiosity or demand decision in order to fulfil some social good.

From school, most people gain the impression that mathematics is a cold, unemotional, cut-and-dried subject in which geometrical theorems are proved by rote methods or algebraic formulae are committed to memory for regurgitation. Only occasionally in school mathematics, as taught traditionally, did the pupil feel even a faint twinge of the excitement, agony and intense emotion which commonly accompany the activities of a creative mathematician - ecstasy when he finally proves his conjectured theorem, despair when he shows it is false. The search for an answer can go on for years. In the case of the so-called Last Theorem of Fermat [3] for more than three centuries mathematicians have been unable to decide whether it is true or false.

A foretaste of this delight and excitement is sometimes experienced in solving mathematics problems in high school. Perhaps it is this which first catches the interest of students. There is satisfaction in obtaining a solution, especially if initially the problem seemed difficult, and joy in producing an "elegant" solution - one which brings all the essential elements vividly before the mind.

> Then felt I, like some watcher of the skies,
> When a new planet swims into his ken
> Or like stout Cortez when with eagle eyes
> He star'd at the Pacific - and all his men
> Look'd at each other with a wild surmise -
> Silent, upon a peak in Darien." [4]

This is the feeling. Evidently many students have a slight experience of it, since in polls among Ontario high school students, mathematics has been declared to be the most popular subject.

However, to keep the research mathematician at work over many years, an additional factor is generally needed. As in most other areas of human endeavour, "Fame is the spur." Though to many people, mathematicians seem to be a strange and alien race, they are in fact human ... at least most of us are, so we like recognition. This can come in two ways: (i) concretely as promotion, increased salary and generous NRC Research Awards, and (ii) in the admiration of our peers concerning our mathematical papers and books.

There is a widespread opinion that mathematics is a young persons discipline - if you have not made your mark by thirty, you never will. The evidence for this is mixed. Certainly, there have been some great mathematicians who have made their most important discoveries in their early twenties and became full professors at twenty-five. However, there have been notable exceptions. Weierstrass' [5] most important work was accomplished after he was forty. R.L. Jeffery [6] was a fisherman till he was twenty-seven, obtained his PhD only at thirty-five and became one of the most distinguished native-born Canadian mathematicians. However, it seems true that if a mathematician has not attained a good reputation among the people in his own field by thirty-five, there is a high probability that he never will. If his work is such as to interest a wider group of mathematicians in other parts of mathematics, he is well on the way to becoming famous; if he is invited to give a one-hour talk at a national or international mathematical conference, he has almost achieved the ultimate. The only higher accolade would be winning the Fields Medal[7].

According to the level of recognition by his peers which he achieves in the ascending scale just described, the mathematician will expect corresponding recognition by his university in increased salary and promotion. At least such has been the situation in North America in the last few decades when the universities have been expanding rapidly. This expansion has accompanied the rapid expansion of population in the West since 1900. Prior to that date, university population was relatively stable, salaries were fixed and promotion occurred generally when the one and only professor died or retired. Since pensions were not common, few retired before death. Then everyone might move up one rung. Such were the conditions in the steady-state which, apparently Canadian universities will approximate for the next few years. The conflict between the psychological expectations to which faculty members have been conditioned during the recent phase of expansion and the probably stringent economic conditions of the Canadian universities will certainly give rise to considerable stress within the system, even if it does not revert to the rigidities of the ninetheenth century.

One of the most obvious signs of prestige in the scientific community is the size of one's NRC grant. The description in C.P. Snow's The Masters of the anticipation before and the depression after the annual announcement of the results of election to the Royal Society conveys in a heightened and exaggerated form, the feelings of research scientists in Canada around March 30 each year when the NRC Operating Grants are announced. Those who are given the highest grants are delighted and are the object of the admiration or envy of their fellow scientists. Few, if any, will admit to others and least of all to themselves that their grant was as larged as they "deserved". After the whole list has been published, it is examined carefully to determine where one's acquaintances and oneself place in this important annual pecking order. In a muted way, the NRC Awards play a role in the scientific community somewhat analogous to the federal government's New Year's Honours List in the community at large.

Undoubtedly, however, the main factor which keeps a mathematician hard at work - not infrequently for fifty or sixty hours a week - attempting to roll back the frontier of mathematical knowledge, is immediate delight and enjoyment in a fascinating activity. Many mathematicians distress their spouses by becoming so engrossed in research that they can find no time for love-making, eating or playing with the children. Mathematical creation is an intense, demanding and ultimately solitary activity. It produces habits and attitudes which deviate markedly from the normal adjusted well-balanced bourgeois mean of Canadian society. As such, it should be neither praised or deprecated. It is what is necessary for the creation of deep and meaningful mathematics.

The Encouragement of Research

Mathematical research should be consciously encouraged and fostered by Canada and we should undertake more explicit efforts to put the product of such research at the disposal of those who can use it for some worthwhile purpose.

Apart from projects involving considerable computer time, research in the mathematical sciences is comparatively inexpensive. One of the briefs submitted to the Study argues "Pure research in mathematics is relatively cheap (only salary of the researcher - no equipment) and represents good value for money spent - it would be false economy to cut back on pure research". That mathematics is cheap is a view which seems to be shared by the NRC since, as can be inferred from Appendix V, the average Operating Grant to mathematicians in the period 1970-1974 was about 45 percent of the average for all sciences and about 33 percent of the grants to chemists!

What does the research mathematician need in order to work well?

(a) Pen and paper;

(b) A library which can be accessed immediately when an idea strikes him and needs to be checked;

(c) A chalk-board in a small congenial room suitable for excited discussion, with a limitless supply of coffee close at hand. (In 1890, J.J. Sylvester felt "a decanter of ripe port at one's elbow" was the ideal catalyst! Coffee is now generally preferred.);

(d) One or more (three to five is optimal, perhaps) fellow mathematicians to talk to, who will be sufficiently interested to criticize half-baked ideas or suggest their own wild ideas of possible alternative routes to the summit.

Currently, only (a) is conveniently and universally available to Canadian mathematicians.

As for (b), there are in Canada ten or twelve mathematical libraries which could be described as Good to Very Good, and none Great to Very Great (to use the scale for rating Burgundy!). Not unnaturally the good ones are on the larger campuses where in toto perhaps 75 to 80 percent of academic mathematicians work. Too frequently, however, the omniphagic centralizing syndrome which seems to dominate most Chief Librarians, means that when he needs to verify his idea or formula in a book immediately at hand, the mathematician realizes that it is over in the Central Library guarded by the latest electronic portcullis.

What test tubes, reagents and spectrometers are to chemists, what lasers and radio telescopes are to physicists - books and journals are to mathematicians. The spectrometers are in the chemist's laboratory. The physicists lives, eats and sleeps near his laser or telescope. The books which the mathematician needs should be immediately at hand. This necessitates a mathematics (or mathematics and science) library with an architectural solution which puts the offices of research mathematicians immediately next to the library. And, of course, as in Princeton's Fine Hall there need to be a couple of soundproofed blackboarded rooms in the library for discussing questionable points with an array of journals and books spread out on a table for immediate consultation, (since often it will be two or more mathematicians who rush to the library to prove they are each right).

It can be argued that books which are commonly consulted by professors of several departments should be housed centrally. This is a valid argument, but it is not applicable to 98 percent of the books which a mathematician needs readily at hand. Who but a mathematician will consult Gaston Darboux's great Traite des Surfaces or Hilton and Wylie's Homology Theory?

As for (c) those of us who experienced the mathematical life of Princeton in its hey-day know what an important role was played by the gracious common-room at all hours of the day, and by the ritual of afternoon tea and biscuits which drew graduate students and staff together for excited discussion. Too often, architects and those who decide the budget restrictions for buildings think of research scientists as monks who have eschewed all carnal pleasures (such as coffee) and who should be isolated in small cells on long corridors designed to make communication as difficult as possible. In fact, the intense ultimate solitariness of the research mathematician can only be sustained by the support manifested through the interest and criticism of his fellows.

The effective realization of desiderata (b) and (c) should be the highest priority of all architects and administrators responsible for creating the physical conditions in which the mathematician works.

The above three points deal with physical, external circumstances and therefore in the final analysis are not absolutely essential for a truly dedicated person. The human spirit can triumph over appalling physical handicaps. However, (d) is <u>essential</u> and it is the hardest condition to achieve effectively in Canada. Nothing worthwhile can happen unless there is at least one really first-rate mathematician. The great giants of the past, Newton, Euler, Gauss and Cauchy, provided the main impulse which initiated the extraordinary upward sweep of mathematics and its applications which we have witnessed in the past two hundred years. They were mostly solitary. Even so, they wanted and needed the stimulus of correspondence with other mathematicians, but better would have been someone to talk to. Gauss wrote wistfully to a mathematical friend - "It is a great stimulus to have students with whom to discuss mathematics. Currently, I have three. One is intelligent, but lacks preparation. Another is well-prepared but not very intelligent. The third lacks preparation and intelligence." If such giants needed personal stimulus then it is even more important to lesser scientists. Even Hilbert was very dependent on interaction with Minkowski and others.

Interaction can be (i) in written form, (ii) by telephone or (iii) by face-to-face encounter. Given the hectic pace of modern life (i) seems too slow and now plays a relatively unimportant role apart from the extensive use of journals in the library. Possibly (ii) is of increasing importance though it is difficut to obtain hard data about its significance. The discovery by Rudvalis [8] of the simple finite group of order $2^{14}.3^{3}.5^{3}.7.13.29$ came almost by chance as a result of a telephone call.

Of course, direct encounter is most effective. This can be provided partially by brief visits, by meeting at large conferences, by intense topical seminars or institutes for one or several days. All of these methods should be enthusiastically recognized and supported financially by universities, provincial governments, the NRC and other federal agencies. However, by far the most effective stimulus to worthwhile sustained research is the existence of a group of excellent mathematicians in a physical set-up which encourages lively daily interaction. The group might consist of one outstanding senior mathematician with several younger colleagues, postdoctoral fellows and graduate students. The minimal critical "mass" is 2 or 3, but there is also a maximal critical mass, perhaps 10, beyond which administrative, financial and personal problems begin to dominate and quality and even quantity frequently decreases. There are a few such groups in Canada but hardly any Great and none Very Great. Even though Canadian universities spend more money than all the rest of the universities of the Commonwealth, mathematically we are still very much underdeveloped. Further, few of such groups as do exist manifest any sense of responsibility to communicate with mathematicians or users of mathematics beyond their own esoteric field. We have still a very long way to go.

Every Mathematician a Researcher?

The main criteria used by the Awards Committee of the NRC is the number and quality of substantial papers published in refereed journals. Since our society is based on the premise that "money talks", a large NRC Operating Grant to a particular staff member enhances his own reputation and that of his department and university. It improves his chances of rapid promotion and increased salary. Certainly the slogan, "Publish or perish" goes too far to portray the actual practice in the average Canadian university. Perhaps "Publish to rise rapidly" more aptly suggests the Canadian atmosphere.

Of course, the only sound motive for publishing is that one has discovered an important or worthwhile new result or new enlightening approach to one or more known results deemed to be of interest to a wider audience than one's students and colleagues. It is a duty to expound such material as lucidly as possible and attempt to have it published and so contribute to the total body of public knowledge. The "dog-in-the-manger" spirit dies hard of course, so often general publication of a result will be delayed in order that its discoverer or his or her co-workers or students can have significant lead-time over groups elsewhere in exploiting the idea. This practice was common enough in the 17th and 18th centuries. It still occurs in highly "competitive" areas of science. However, most scientists view such action as immoral, as a betrayal of the basic commitment of the community of science to discover and disseminate truth.

What of those of us who have no ambition to "rise" rapidly? Need we publish or do research? In order to retain our own respect, the respect of our colleagues and the respect of our students, we must work constantly to keep abreast of what is going on in mathematics. This is more than a full-time occupation. For most of us, the only sure way of keeping mathematically alert is to organize our reading and study with a view towards publishing new mathematical results, an exposition of some broad area of mathematics or a discussion of a significant application of mathematics. Indeed, the mathematics community must find ways to encourage the publishing of studies of the interrelation amongst different parts of mathematics and of new applications of mathematics to other disciplines. There is no simple[9] formula of how this can be done and at the same time decide upon and maintain reasonable standards.

It would appear that the mathematicians of the USSR have been much more successful in these matters than we in North America. Kolmogorov is active in the reform of curriculum for high schools. I.M. Gelfand, whom many would regard as the greatest living pure mathematician for his work in group theory and generalized functions, has a seminar on biomathematics. For many years, Pontrjagin has conducted a seminar in Control Theory. It is almost inconceivable that any of the leading mathematicians of North America would condescend to write school textbooks. Gelfand is editor of a Library of High School Mathematics and has co-authored a charming little book on Functions and Graphs for Grade Nine.

To summarize our main convictions: every academic mathematician should actively attempt to keep abreast of his own field, he should explore its ramifications into other branches of mathematics and other sciences and he should keep before himself the aim of publishing at least some of the results of his study in order to discipline and focus his own efforts and in order to subject his ideas to the appraisal of his informed colleagues. He must do all this to keep academically alive and in order to help the mathematics community fulfill its role in the university and in the wider community.

Granting Policies

At the moment of writing, the Canadian Government [10] is in the process of reorganizing the structure of the bodies principally responsible for awarding grants for university research. When we referred to the NRC in the past we meant the National Research Council but, in the future, the Granting Council for Natural Sciences and Engineering.

We have already alluded to the key role which the NRC Operating Grants play in establishing prestige among Canadian scientists. Indeed, so much so that a committee, which had been asked to assess the state of chemistry in Ontario, simply calculated the total NRC grants to the various Ontario chemistry departments and rated them in direct proportion to these totals. Many informed observers of chemistry in Ontario felt that several of the conclusions reached by the committee on this basis were quite fatuous.

Nonetheless, there is much evidence that the procedures used by the NRC for supporting research have been at least as satisfactory and fair as those in most western countries. Each year, the federal government determines the total funds available for the support of in-house research in the NRC Laboratories and also the funds available for research in the universities. A committee of the NRC, on the advice of the Officers, allocates this total among its 16 discipline Grants Selection Committees. (See Appendix V.) The Committees - whose members serve without pay - study the applications, decide which are most worthy of support and allocate the funds at their disposal. Thus, the precise amount of an Operating Grant is decided by a committee of academics in one's own field. Hence the procedure is frequently referred to as the "peer group" method. Clearly, it is much better that these decisions be made by persons acquainted with the subject-matter of the proposed research than by a general-purpose bureaucrat who would be completely at sea in evaluating the application.

However, there are two constraints on the present system which may make it less than optimal:

(1) In order to be fair to all sciences, the NRC has a set of rules which ostensibly applies uniformly to all disciplines. Since the NRC has always been dominated by the experimental sciences, these rules are not necessarily the best possible for encouraging the mathematical sciences or such subjects as theoretical physics and theoretical chemistry. Symptomatic of this perhaps was that in 1958/9 only $50,000 was given to support mathematics. Even though

the NRC officials have been flexible and very sensible in interpreting their rules, the printed regulations create a mind-set in the committees and in the attitudes of researchers which is not always the most creative.

(2) More important is the value-scheme which permeates a discipline and is the chief factor in determining the decision of the individual Grants Selection Committees. In order to justify their decisions to the scientific community and to themselves, the members seek "objective" criteria. The easiest to define and employ is the quantity of papers published by applicants in refereed journals. Though individual members of the NRC Mathematics Committee stoutly maintain that this is not the only criterion which they employ, seldom have they been able to explicate any others which they use or which are in any way as significant as this particular factor.

The possible and actual evils of this process are patent. Since "money speaks", researchers are delighted when they are awarded large grants. Deans and Presidents view them with enhanced approval. In many Canadian universities, the income from NRC Operating Grants is a predominant factor in financing Schools of Graduate Studies and Research and so maintaining the quality and reputation of the university. Perhaps the clearest sign of fame for a Canadian scientist is the size of his NRC Operating Grant. The larger his grant, the more graduate students, postdoctoral fellows and research associates with whom he can surround himself in order to gather as many rosebuds as possible from the particular garden he is cultivating. The system involves positive feedback. The larger the grant, the more workers, the greater the prestige, the more papers and the larger the next grant.

This process is called "supporting excellence". It is justified by the argument that most funds should be given to the ablest (more productive!) scholars who should then be given a free hand and not inhibited by annoying and pointless red tape. Gerhard Herzberg[11] is certainly one of the foremost and one of the most convincing exponents of this argument. The argument would be unassailable if there were any method - even moderately good, not necessarily infallible - to recognize a Nobel laureate twenty or thirty years before he wins the Nobel prize! Unhappily, there is not. Many of the positive feedback research loops just described end up in a cul-de-sac. A large number of able postdoctoral fellows and postgraduate students become trapped in an essentially unproductive and frustrated pattern of research for many years. Currently, a striking result of the process we are describing has been what most Chairmen of Mathematics

Departments regard as the excessive overproduction of PhDs in algebra in Canada and North America generally. Between October 1 and December 31, 1974, one Canadian mathematics department which will have no opening on its staff in 1975/6 received 71 unsolicited applications from PhDs (15 Canadians, 56 Non-Canadians) of whom 30 were algebraists (6 Canadians, 24 Non-Canadians). Only 9 were in statistics or applied mathematics.

Certainly, the current "PhD crisis" has not been caused solely by the peer judgement system as operated by the NRC. There are many other factors - but this is an obvious one and it may well be the only important factor which is susceptible to control. A common defence of the present NRC system is that the only "conceivable" alternative - rigid bureaucratic controls - would be much worse. This is agreed. Indeed the views of the Lamontagne Committee [12] seem quite uninformed at precisely this point.

(i) The Committee seemed unaware of the existence and basic role of mathematics in our society since its Report makes no significant reference to mathematics;

(ii) It displayed little understanding of the delicate and subtle intermeshing of a wide variety of factors which are necessary to encourage lively and significant research. The Committee ingenuously assumes that real progress can be made in research and development by administrative fiat and organizational reform. This is most unlikely.

By far the most important step which can be taken is the transformation of attitudes and expectations of Canadian research workers. To put it succinctly and therefore to over-simplify, the academic community must accept responsibility not merely for "research" but also for "development". Further, the granting policies of the NRC should be designed to encourage such a change of attitude.

In Chapter IV we warned against entertaining the expectation that the fundamental changes which are needed in the attitudes of school teachers towards mathematics could be effected overnight. Here again a similar warning is appropriate. The major reorientation of the dominant spirit of the mathematical community which we propose can only result through a gradual process of education and discussion among the creators, teachers and users of mathematics. Such a discussion took place already in connection with the preparation of the many briefs submitted to the Study and was particularly pointed out in the eight Seminars and during the meetings of the Joint

Committee. Indeed, it began in earnest three or four years ago in the Notices of the American Mathematical Society and the American Mathematical Monthly.

The decisive role which research grants to universities from external bodies can play in deforming the character of the academic community, has been described in frightening and convincing detail by R. Nisbet. His important work[13] analyzes the history of the university in the U.S.A., showing that the increasing prestige of masters of grantsmanship, and the rise of research centres, within but not responsible to the university (because their finances derived from external grants) has caused a tragic distortion of the value-scheme within the U.S. academic community and effectively destroyed public trust in university professors. Happily, the process of degredation which Nisbet describes is only at an incipient stage in Canada. But it is surely not necessary for Canadians to commit all the errors of our colleagues in the U.S.A.

Specific Suggestions

Input to the Study was weaker on questions related to research policy than on almost any other topic. In particular, there were only a few vague responses to our question as to the reasonableness of the distribution of NRC awards in the year 1973/4 among the various sub-disciplines of the Mathematical Sciences as set forth in Appendix VIII. To have a neat bureaucratic solution to the problem of distributing funds in a manner which would be in the best national interest, one would need to be able to decide whether differential equations, multivariate statistical analysis or artificial intelligence, for example, needed strengthening or were already over-developed. Since only five persons ventured even a vague comment along these lines, it would seem that the "best national interest" is an undecidable question when it is brought down to this detailed realistic level. There is no neat system by which some bureaucrats sitting in Ottawa can meaningfully divide the research pie. Possibly the system as it has evolved in the NRC is the best possible, or at least a reasonable approximation[14] to the optimal solution. Perhaps the scope of the fields covered by Code Numbers 0001 to 0069 of Appendix VIII is too broad to be encompassed meaningfully by a single committee. It has been suggested, for example, that a separate committee for Probability and Statistics is needed for fields 0040 through 0056 to ensure that statistics - particularly applied statistics - is supported adequately. However this proposal was countered by the argument that we should resist the institutionalizing of ill-defined divisions between the mathematical sciences.

Since in the course of the Mathematics Study we fell far short of being convinced that we had input from a representative sample of those most active in research in the Mathematical Sciences, the suggestions which follow need widespread informed critical discussion before they are implemented. Some of the suggestions could be implemented by the NRC, some by the mathematical societies - particularly the CMC - and some will require cooperation of industry and various branches of government. They should be considered in the light of the previous remarks about the conditions necessary to encourage research in mathematics.

(1) Emphasize Meetings and Communication

Particular mathematics departments, or groups of departments, or the mathematical societies could take more initiative than they have in the past to organize short topical research seminars or conferences. There are several possible models: meetings of the Institute for Mathematics and its Applications in the U.K.; seminars financed by the National Science Foundation in the U.S.A.; the study weeks at Oberwohlfach in West Germany. A three-year rolling plan of topics which should be encouraged for one reason or another in Canada could be prepared by the Canadian Mathematical Congress or the Council for Mathematical Sciences (if that body comes into being).

(2) Change Emphasis From Individual Grants to Block Grants

This suggestion runs counter to a basic tenet of recent NRC policy. That policy may be particularly appropriate in an experimental science in which a major physical installation has to be created and maintained over several years before significant results can be achieved by one individual supported by a variety of assistants. However, this consideration is not appropriate to mathematics. A block grant might be awarded to a consortium in one of more universities which puts forward a considered proposal aimed at excellence (i) in a particular discipline or (ii) in an interdisciplinary area (either between sub-disciplines of mathematics or between a mathematical and another science). Type (ii) should be given preference. The program of Negotiated Development Grants undertaken by the NRC a few years ago contained this idea in embryo. However, the original "pump-priming" conception of the NDG's proved unrealistic. The hope that a university could move some area of research to a significantly higher level within three years and then find that it had become financially self-sufficient, was actually realized in only a few instances. Perhaps if the universities had continued in the expansion phase of the late 60's, the NDG's might have been more successful. A key problem is that of lead-time. To develop a centre of excellence in almost any of the hard sciences requires a minimum of ten years with a solid commitment of adequate continuing financial support. In this context, we remark that the current system of three-year Operating Grants is an improvement over the previous system in allowing research to be planned more rationally.

(3) The "Development" Aspect of the Mathematics R and D Effort Should be Encouraged.

Recall that we are using "development" in the context of mathematics in a somewhat unusual sense. By it, we mean almost any effort to get the academic mathematicians out of their isolation. For example, in the writing of papers dealing with new results in some narrow part of mathematics, an effort should be undertaken to expound its relation to other parts of mathematics. (If it has no relation to anything else, it is almost certainly of little interest and the wisdom of publishing it would be very doubtful), surveys of substantial areas of research in a form which makes the results accessible to others; attempts to apply known mathematical theorems or methods to other sciences and engineering; the organization of problem-solving Seminars such as those held at the Mathematics Institute in Oxford between mathematicians and individuals in business, industry or government. The NRC[15] could call for proposals along such lines and announce the intention that within some fixed time, say five to seven years, a certain proportion, say 50 percent, of NRC grants in the Mathematical Sciences would be allocated to "development". The application forms for NRC grants should be revised to require researchers to describe their efforts in "development". This is of course a region into which we would have to proceed gradually, feeling our way very carefully since we would be on quite new ground and the possibilities of charlatanism and grantsmanship *par excellence* would certainly abound in such terrain. However, by moving with due speed, it should be possible within three to five years to evolve a set of working rules which would obviate gross abuses and prevent too much nonsense.

(4) Decouple Graduate Support from Research Grants

A graduate student who is paid from an NRC Operating Grant is supposed to be helping the principal investigator on his research. This principle is probably honoured more frequently in the breach than in the observance. Certainly, among mathematicians! However, even if a graduate student does not work directly on a topic of one's research interest, the presence of graduate students in the department usually heightens the level of mathematical activity and encourages research. Further, the total research enterprise can be maintained only if able new researchers are trained. So the financial support of graduate students by the use of NRC funds is clearly justifiable.

Perhaps the most effective support of graduate students would result from a system by which a block grant were given to the Department, or to all the recipients of Operating Grants to be administered by them jointly. The possible effects of any such change should be considered very carefully before it is instituted, and its introduction should not be effected abruptly but rather with due warning and over a period of two or three years, taking into consideration prior moral commitments to individual students. In the experimental sciences, the coupling of graduate student support and Operating Grants may be fully justified, since often the graduate students provide cheap labour for the holder of the Grant, learn the research techniques in the process, and are ultimately rewarded for their efforts with a PhD. It is a valid form of apprenticeship.

(5) Special Treatment for Mathematicians in Isolated Centres

Thought should be given to making the NRC rules more flexible in ways which would benefit mathematicians in small or isolated centres. It would be a bit dicey defining "small" and "isolated" in this context but satisfactory though arbitrary rules could probably be agreed upon. For example, a "small" department might be one in which 5 or less members are awarded NRC Grants; "isolated" might be a university such that within a radius of 200 miles there are at most 10 mathematicians who hold NRC Grants. Mathematicians in such centres could be given grants for travel, long-distance telephone calls and photocopying which are twice the average of such grants to other mathematicians.

(6) Basic Grants to Many Mathematicians

Research mathematicians need funds - (i) to visit other mathematicians, (ii) to bring other mathematicians to their campus to share ideas, (iii) for long-distance telephone calls to check a doubtful point, and (iv) to cover page-charges for their published papers. Possibly a standard grant could be decided upon to cover these basic needs. To obviate NRC paperwork, it could be given as a block grant to be administered by the Department 16 or jointly by the recipients.

(7) The Role of Postdoctoral Fellows (PDF)

The following figures seem so strange as to be incredible. They purport to give the total number of postdoctoral fellows in the mathematical sciences in Canada and the U.S.A. in the Fall of 1971 and 1972.

	1971	1972
U.S.A.	261	228
Canada	115	153

The figures for Canada were reported to the Study directly by mathematics departments and should be quite accurate. The figures for the U.S.A. are from the report on Graduate Science Education, Fall 1972. (National Science Foundation 73-315) and should also be correct. If so, they disprove a widely held opinion that everything in Canada is one-tenth of the same thing in the U.S.A. Also, while the number of PDF's in the U.S.A. decreased from 1971 to 1972, there was a marked increase in Canada.

We do not know what proportion of the PDF's in Canada were graduates of Canadian universities. But recalling that in 1972 there were only 38 new positions in mathematics in the universities and there were 88 new PhD's, it is not unreasonable to guess that the difference (153-115 $\simeq$38) includes a fair number of the 88-38 $\simeq$ 50. If so, then given the pattern of graduate study which was discussed in Chapter VI, it is probable that they spent the PDF year examining in greater depth the topic of their PhD thesis in the hope of publishing and gaining an academic position. Since it seems probable that there will be very few new university positions in the next five to ten years, such hopes would almost certainly prove illusory.

However, we have emphasized repeatedly in this report that there is a pressing need for mathematicians to interact with scientists in other fields. So the idea has been proposed that mathematical PhDs who do not find other suitable employment and apply for a PDF should be granted a fellowship for two years only in order to gain a working knowledge of a subject other than mathematics in which relatively sophisticated mathematics can be applied. To implement this idea will necessitate a basic change in the working-rules of the National Research Council and the Medical Research Council.

An activity which PhD's in North America seldom take up is teaching in high school and community colleges. We have already noted that Alexander Wittenberg remarked that he taught for ten years in an ordinary Swiss high school in which one mathematics teacher had a Master's degree and all others a PhD. Weierstrass [5] taught high school.

(8) Exchange Between Universities, Industry and Government

This idea has already been mentioned in Chapter III but should be recalled at this juncture as providing a source for important and challenging research problems and opportunities for outreach by mathematicians.

(9) Representation of Mathematicians on the NRC

The distinguished Canadian mathematician, Max Wyman [17] was a member of the Council of the NRC. As far as we can determine, he was the only mathematician who has played a significant role in governing circles of the NRC or the Canada Council (which also awards grants to mathematicians). This record is appalling in view of the fundamental role of mathematics in undergirding the other sciences and of the very special character of the problem of encouraging mathematics research. The Deputy-Minister of the Ministry of State for Science and Technology, in his role as Chairman of the Tri-Council Co-ordinating Committee, should ensure that mathematicians are properly represented at the highest levels of decision-making for government policy concerning research support.

....

It is obvious that the above nine suggestions are not of equal importance and that their implementation would involve quite different agencies. It is urgent that all of them, and doubtless many other ideas, be seriously considered, that decisions be made and that action ensue.

To get this ball rolling, it is recommended that the Ministry of State for Science and Technology institute a review of granting policies of the federal government for the support of research to determine

(a) how the mathematical community can be represented more adequately at the senior levels of decision-making for federal support of research,

(b) how the total financial support by the Federal government of research in the mathematical sciences can be sustained at an adequate level.

(c) whether it would contribute more effectively to the development of the mathematical sciences in Canada if the current practice of the NRC, of awarding most grants to individual researchers, were gradually modified to emphasize somewhat more strongly the support of projects which bring Canadian researchers in the mathematical sciences into active interaction.

Notes - Chapter VII

1. From Alfred Tennyson's famous poem *Ulysses*.

2. Constance Reid, who is not a mathematician, has written a charming and perceptive biography of David Hilbert who was a dominant mathematical figure at the turn of the century. In his lecture at the International Mathematical Congress in Paris in 1900, he posed a series of unsolved problems which have played a major role in guiding the development of the whole of mathematics research for the first half of the twentieth century. Reid's biography is called simply *Hilbert*. It was published in Berlin and New York by Springer-Verlag in 1970.

3. On the margin of a book he was reading, Pierre de Fermat (1601 - 1665) wrote that he had found a marvelous proof, which was too long to write in the margin, of the assertion that if n is a positive integer, greater than 2, there are no triples of integers (x, y, z) satisfying the equation $x^n + y^n = z^n$.

It is known to be true for $n = 3, 5, 7$ and many other values but no general proof has yet been obtained. This is the most famous unsolved problem in mathematics. The classic problems of "squaring the circle" or "trisecting a general angle" by means of a ruler and compasses alone, for which many amateurs advance new "solutions" almost every year, were proved to be insoluble in the 19th century.

4. From Keat's poem "On First Looking into Chapman's Homer".

5. Karl Weierstrass (1815 - 1897) was a very powerful mathematician whose work had a great influence on the theory of Complex functions. He taught in high school for many years - not only mathematics but also gymnastics, science and writing. There is a story that one morning he missed an eight o'clock class; this was so unusual that the director of the school went to his room to discover whether he was ill but merely found him working eagerly at research on Abelian functions, which he had started twelve hours before and continued through the night. He was unaware that daylight had come! He became a university professor at the age of 49 and continued actively at research into his old age.

6. Ralph Lent Jeffery studied and taught at Acadia University in Nova Scotia. He was Head of Mathematics at Queen's University from 1943 to 1960. At the age of 85 he was revising his treatise on the Theory of Function and actively lecturing on mathematics!

7. The Fields Medals have been awarded every four years since 1936 to outstanding young mathematical researchers. The medal is cast by the Canadian Mint. The award was established in the will of J.C. Fields who was an algebraic geometer at the University of Toronto. For mathematics it is the closest equivalent to the Nobel Prize. There is a story that Nobel was jealous of the fact that the contemporary mathematician, Mittag-Leffler, had frequent invitations to play whist with the Swedish king and to make sure that he would never win the Prize, Nobel directed that there should never be a Prize in mathematics!

8. Arunas Rudvalis, A New Simple Group of Order $2^{14}3^{3}5^{3}7.13.29$; Notices of American Mathematical Society, January 1973, Abstract No. 143.

9. The following rather surprising passage from the Foreword to the Revised Edition of 1960 of A Book of French Wines by P. Morton Shand (Revised by Cyril Ray), Penguin Books, 1964, raises vividly

the universal problem of standards. "The craft of criticism ought never to be thought of as confined to the fine and applied arts and literature with a capital L. Wine is as much an incessant human striving for perfection. For balance - the supreme informing quality which distinguishes all great wines - is only another aspect of form, and like poetry, sculpture, music, or architecture can from time to time achieve a measure of sublimity. All good criticism is the expression of the same comparatively rare type of interpretive intelligence, and therefore in essence one."

10. It seems unlikely that the new legislation will come before Parliament in 1975 even though its imminence was announced early in 1974. Unhappily, such delay and indecision on the part of the government and the resulting uncertainty among scientists has been only too typical of the Canadian approach to science policy during the past two decades.

11. Dr. G. Herzberg is a member of the permanent research staff at the National Research Council. He was awarded the Nobel Prize for his distinguished work in molecular spectroscopy, in 1971.

12. See Note 5, Chapter VI.

13. <u>The Degredation of the Academic Dogma: University in America, 1945-70</u>; Robert A. Nisbet, New York, Basic Books (1971); John Dewey Society, Lectures, Number 12.

14. A widely respected applied mathematician who has served on the Mathematics Grants Committee of the NRC submitted the following views to the Study:

"I believe that the present method of support of basic research in all the sciences through individual operating grants from N.R.C. is a good one. The only important exception I would make is that I believe that the support of graduate students should be separate from the research grant system much more than it is now.

"At least in the mathematical sciences, I think we would benefit from the transfer of a substantial portion of the funds from operating grants to scholarship funds for graduate students. Unlike the situation in the experimental sciences, a graduate student is seldom an important assistant in carrying out a faculty member's research project. Thus it is entirely possible to separate the questions of graduate student support and of research support for a faculty member in the mathematical sciences. This separation would have several advantages, including the following

(i) It would be easier to control the number of students who undertake graduate work in mathematics in Canada.

(ii) There would be more freedom to experiment with graduate programs which have little or no research component.

(iii) The criteria for awarding grants and the legitimate uses of grant funds would be clearer to Grant Selection Committees and grantholders alike. I would hope that there would be adequate funds to provide a reasonable amount of money for travel, photocopying, publication charges, and computing for all those actively engaged in research. This amount of money would not be large and could be adjusted somewhat according to other demands on government finances. In addition, I would like to see much larger amounts of money available to those outstanding researchers who were able and willing to use these amounts. These funds could be used to pay postdoctoral fellows, visiting scientists, and so on. I think the support for outstanding researchers is even more important than the modest support of the great mass of researchers; however, I think both types of support are valuable and should be maintained."

15. One correspondent, very keen to encourage university mathematicians to relate more actively with users, is not convinced that the NRC is the best agency to encourage this. He writes

"I do not think that N.R.C. grants are the best vehicle for the encouragement of more applied research. I think that departments of government which would like to see research done in their own special areas should expand their research grant and research contract programs. I have in mind here grants and contracts which would be much more mission-oriented than N.R.C. grants. Presumably such grants would leave much less freedom to the researcher to go off in directions of his own. Perhaps the Ministry of State for Science and Technology should have the responsibility of deciding the appropriate division of the nation's resources for research between the N.R.C. type of basic research and the mission-oriented research of other departments. Finally, I should say that we should be careful not to involve universities in research tasks which are not appropriate to their capabilities. The university does not have the solution for all society's problems and we should not pretend that it does. Government can get advice from its own servants and from special commissions which devote their full time to a question. Development projects will usually be done better in industry than in a university."

16. One correspondent writes "I would prefer all grants to go to individuals. Otherwise I am sure that some individuals will not be given their due because of department politics."

17. Max Wyman was Professor of Mathematics and, more recently, President of the University of Alberta. He is a former President of the Canadian Mathematical Congress.

Chapter VIII

The Reward System

"He played the recorder. That was was the reason we hired him."
Interviewer, *"Because he played a recorder!"*
"Yes, we thought that would be nice."

"For most members of the profession, the real strain in the academic role arises from the fact that they are, in essence, paid to do one job, whereas the worth of their services is evaluated on the basis of how well they do another ... Most professors contract to perform teaching services for their universities ... however, when they are evaluated it is in terms of their research contributions to their disciplines."[1]
- T. Caplow and R.J. McGee

The first quotation illustrates rather pointedly that human decisions - even in universities - are not always as rational as a Systems Analyst might sometimes postulate for his models of social process! The second quotation alludes to the publish-or-perish pressure under which many academics live. Such pressures are much more intense in the outstanding universities in the United States than in most Canadian universities. Nonetheless many Canadian academics experience an uncomfortable tension between the demands of their students, the expectations of their provincial paymasters, their commitment to their discipline and their ambition to be recognized as sound scholars by their peers. The position of the mathematician in industry or government can be even more difficult and frustrating - especially if his boss is suspicious of mathematics but at the same time expects of it miracle solutions to problems which have no neat solutions.

Mathematical Prestige

There can be no question that the discovery of significant new mathematics is of central importance to the whole mathematical enterprise. This is the Mount Everest which challenges everyone in the mathematics community. This challenge provides the <u>elan vital</u> which has created mathematics as we know it today and therefore undergirds the whole of our technological society. It is therefore appropriate that the highest honours and prestige should be accorded to the creators of important mathematical theories. However, not every mountaineer spends his time climbing Everest and, indeed, it is the plain-dwellers who grow the food which nourishes the mountaineers!

In our view, it has been a serious defect of the mathematical community throughout the world but especially in North America to accord significant honour to no activity except the creation of new mathematics. Thus, writing expository books and articles, teaching mathematics, or communicating empathetically with the users of mathematics have been awarded almost zero prestige compared with research. The predominance of this one value within the mathematics community has had an extremely bad effect. Since probably less than one in ten mathematicians create new mathematics which is accorded a permanent high valuation, nine out of ten have a conscious or unconscious sense of failure. Thus personal satisfaction is at a low ebb among mathematicians over forty years of age.

The predominance of this one value has brought much personal despair. However, its public effects have been even more serious. It has caused the mathematics community to turn in upon itself and become self-absorbed. This predominance has been the chief cause of the failure of mathematicians to be out-going in dealing with the users of mathematics.

Absorption in their discipline is not peculiar to mathematicians. This was made clear by Caplow and McGee in their well-known book and is illustrated by the second of the quotations which heads this chapter. They amass much evidence for their contention that prestige is the main factor controlling academia. It is almost impossible to gain prestige if you took a PhD at a non-prestigious university. You cannot gain prestige merely by writing the most important paper in your subject for the past thirty years. Until a prestigious authority in your field (who, necessarily will be a full professor at a prestigious university - California, Harvard, Princeton, the Sorbonne or Moscow) recognizes it as significant no one will consider it worthwhile to undertake the effort of grappling with difficult new ideas.

Even if one is not under internal or external pressure to publish - and such pressures are certainly much less in Canadian than in major American universities - in order to maintain self-respect and the respect of colleagues and students, it is necessary to keep abreast of one's discipline. This can be enormously demanding in time and energy. A young Associate Professor of biochemistry estimated that he works eighty to ninety hours per week, eleven months of the year to maintain his good research reputation. His family gets very little of his time. He lectures only forty hours a year, but this is much compared with biochemists elsewhere. Many research mathematicians would share his feelings - and their spouses would resignedly admit that the syndrome is only too familiar to them. It is single-minded intensity of this order which has created modern science as we know it. The pattern of life of such men and women is totally unfamiliar and almost unimaginable for most people in our society. Yet, unless the staff of a university has a solid core of such scholars - passionately committed to keeping abreast of and extending scientific knowledge - the university will quickly decay into mediocrity and intellectual torpor. A country without such men and women will soon be out of touch with the modern world.

Certainly the mathematical sciences were created in the past by, and depend for their vitality in the present and future on, men and women totally committed to research. They should be honoured for their contribution.[2] However, they work with extraordinary zeal because they enjoy doing so. There is no logical reason why they should be paid high salaries, though it is the duty of governments and university administrators to do whatever is reasonably possible to provide them with favourable working conditions.

However, since the prestige of a university within the academic community depends chiefly on the quality of its scholars, the best are in great demand and the functioning of the Academic Marketplace has caused their conditions of employment to improve rapidly during this century. This has been linked with the population explosion and general exponential growth on all sides. University contracts for able researchers rightly include minimal teaching responsibilities - usually no routine teaching, with the option of offering graduate courses if they feel the desire to do so. However, as Charles Dickens already noticed during his visit in 1842, the spirit of egalitariansim in the U.S.A. and Canada is so deep-seated that no one in North America will admit that anyone else is better than themselves. Most academics think they merit the same salary and working conditions as the ablest researcher.

To justify this, however, we think that we have to be "productive". When we come to the crunch the only "objective proof" of "productivity" is the number of pages written. So it has been since 1900 in North America. The search for prestige by universities and the concomitant pursuit of prestige by academics thus resulted in a "productivity" fetish or idolatry. The unhappy consequences of idolatry, which were clearly enunciated when the Ten Commandments were promulgated on Mount Sinai, have followed ineluctably.

The Alternative

The positive route forward is to explicitly repudiate this obsession with research productivity within the discipline as the one and only value. For each discipline, it is the key value in the sense that it characterizes that particular discipline. However, for the mathematical sciences equally important are (i) teaching, (ii) exposition, and (iii) communicating with the non-mathematician.

Since these three activities have been discussed at length throughout this Report, it should be sufficient here merely to recapitulate the few salient points.

Teaching of mathematics enables students to develop one of the highest capacities distinctive of the human species. Mathematizing should result in joy and satisfaction. A primary duty of the mathematical community is to enable all men and women to experience this joy up to the limits of their capability. To accomplish this primary duty will require as much hard work and creative imagination as the most demanding frontier research.

Exposition is essentially a synthetic activity. Bourbaki's great work is a stellar example. Good exposition illuminates the importance of particular theorems or results and brings them together in a meaningful whole. It excites in its audience new insight, power and delight. It opens up new vistas of possible application of mathematical theory - power. It brings hitherto disjoint and unconnected facts into illuminating interaction - insight.

Communication with the users of mathematics demands on the part of the mathematician some acquaintance with the technical vocabulary of other fields, an unselfish out-going thrust of personality, and great patience in helping economists or biologists bring logical order into their complex data.

All these activities are as important to the mathematical community as research. They should be given equal prestige, and be rewarded as such. University Presidents, Deans and Chairmen of

Departments should require that evidence of such activities be evinced by departments of mathematics. We therefore recommend

in view of the changed role of mathematics in our society, that the officers of Canadian universities should provide increased incentives for academics in the mathematical sciences to make greater efforts to communicate on a professional basis with the "users" of mathematics.

However, it is not only mathematicians whose attitudes need to change. It is equally essential that government, business and industry learn how to make more effective use of mathematics. This will happen only if the incentive structure changes. Top management must reward long-range thinking and planning. Relevant mathematical competence must be sought out and given increased prestige and status.

Notes - Chapter VIII

1. Both quotations are from The Academic Marketplace by Theodore Caplow and Reece J. McGee - an anatomy of the academic profession; Anchor Books, Doubleday and Company, New York, 1965. (Originally published, 1958).

2. One correspondent comments, "I think that 'enjoy' may be the wrong word. For some people science is more an 'addiction' than something they 'enjoy'."

Chapter IX

A Canadian Institute of Applied Mathematics

"To be, or not to be: that is the question:
Whether 'tis nobler in the mind to suffer
The slings and arrows of outrageous fortune,
Or to take arms against a sea of troubles,
And by opposing end them?"
- from Shakespear's "Hamlet"

The complexity and ambiguity of Hamlet's situation provides an apt parallel for the situation produced by the variety of advice and ideas about a Mathematics Institute which have been pressed upon the Study. The very first brief we received consisted of a comprehensive and closely-reasoned argument from Professor F.H. Northover [1] advocating the creation of an Institute which would serve as an active bridge between the users of mathematics and university mathematicians for all manner of applied mathematics. More recently, Professor Israel Halperin [2] circulated a proposal for the immediate establishment of a Mathematics Institute offering ten or more postdoctoral research positions for recent able young Canadians in pure and applied mathematics. A wide variety of purposes have been suggested for such an Institute. There are many models to choose from in Japan, France, Denmark, the U.S.S.R. and the U.S.A. There are mathematics divisions in the Institutes for Advanced Study in Princeton and Dublin.

The idea that Canada needs a Mathematics Institute is not new. Leopold Infeld [3] advanced a proposal for an Applied Mathematics Institute before 1954. In 1962, Queen's University offered to host a Mathematical Institute. However, largely because of in-fighting among the various university mathematics departments for the meagre funds which the federal government was putting into the support of mathematics [4], these previous initiatives proved abortive.

The Need

Much greater efforts must be made to bring about fruitful interactions between university mathematicians and the users of mathematics in Canadian business, industry, government and science. This message was forcefully brought home to us in dozens of briefs and in all of the seminars. It must be regarded as one of the principal findings of our investigation. However, there are a number of formidable obstacles which will make it difficult to accomplish this objective within the existing institutional framework.

The most vexing problems that arise in the biological sciences, in the social sciences, or in the decision-making arena are very seldom formulated in precise mathematical language. In order to be really useful, therefore, the mathematician has to be willing to learn a good deal about the non-mathematical context of the problem. This takes much time, effort and patience. It also requires an attitude of mutual confidence and respect between theoretician and practitioner, so that a productive dialogue can begin to take place, making it possible for a practicable solution to emerge. For the success of such a venture, it is clear that a high degree of personal commitment will be necessary, extending over a considerable period of time. While this can be done and must be done within individual Departments of Mathematics in Canadian Universities, it will not happen overnight, given the attitudinal inertia which is so characteristic of institutions generally. The nature of the reward system within universities will make the transition doubly difficult, since it tends to reinforce existing modes of behaviour at the expense of other, more adventurous undertakings. On the other hand, a single decisive action could start the entire process going.

What is needed is a new locus of activity, with independent funding and without any other conflicting goals, to act as a pacesetter. We believe that an Institute of Applied Mathematics, focussing its attention exclusively on this question, would set an example for the entire mathematical community. Scholars attached to such an Institute could channel their undivided energy into the task of discovering the most promising avenues by which mathematical knowledge can be brought to bear on real-life problems of national importance: problems of transportation, communications, ecological impact, population distribution, and energy policy to mention just a few possibilities. By bringing together like-minded mathematicians who are presently scattered in universities and other centers across Canada, the new Institute could offer a wide range of experience and expertise that cannot presently be found in any single institution in Canada.

Previous proposals for a Mathematics Institute conceived it primarily as a vehicle for the promotion of excellence within the field of mathematical research proper. The new element which has been introduced as a result of the Mathematics Study is the fact that such an Institute, as outlined above, will have a very direct role to play in furthering national goals.

Most of the departments of mathematics in Canadian universities are dominated by pure mathematicians and therefore, to redress the balance between pure and applied mathematics, there needs to be strong institutional support for Applied Mathematics. Since the universities are unlikely to hire many mathematicians in the immediate future, the best possibility for creative action is to start an Applied Mathematics Institute.

Hamlet finally acted. In the present political circumstances, it is unlikely that the Government will act without a positive, precise recommendation from the Ministry of State for Science and Technology.

Possible Functions

Many different functions for a Canadian Mathematical Institute have been proposed. Too many. No one institution could effectively fulfill all the purposes that have been suggested. The members of the Joint Committee for the Study attach different importance to the different proposed functions. Nor is the Committee privy to the priorities of the federal government which are relevant to deciding among them. We must, therefore, attempt to set forth as clearly as possible, a number of alternatives and urge the government to move quickly.

Conceivable functions for a Mathematics Institute include the following.

1) To conduct research in some or all of the mathematical sciences both pure and applied. This is the function suggested by the Steklov Institute in the U.S.S.R. and by the Institute for Advanced Study in Princeton. It is consistent with one of the aims of the N.R.C. - to encourage fundamental research.

2) To promote the rational and responsible use of computers. Just as persons in agriculture, industry and various departments of the provincial and federal governments have obtained valuable technical advice from various in-house laboratories of the N.R.C., equally great value would be derived from an agency which provided authoritative advice on the use of computers in solving practical problems.

There is always the question - will a computer be of any real value for a particular problem? If so, what? Frequently, there remains the need to devise an effective and economical algorithm to solve the problem. To accomplish this often requires the deployment of a wide variety of mathematical ideas and techniques. It is generally admitted that in industry, government and medical research in North America, many millions of dollars are wasted on useless or uneconomical computing.[5] Currently, in the main, the persons most available to give advice are employed by companies which sell computers or computing services and have a vested interest in proliferating waste.

3) To study survey methodology for human populations. There are comparatively reliable methods - of the Gallup-pole type - for quickly surveying public opinion. However, in our consideration of the problem of estimating manpower supply and demand, it became apparent that there are no adequate techniques available anywhere for obtaining accurate information on a wide variety of economic and social indicators necessary for public policy. Often, large sums are expended to collect data which is never analyzed, or which is of dubious validity, or which is analyzed too late to be of other than historical interest. A serious study of the statistical methodology and operational procedures for surveys of this kind needs to be strongly influenced by other mathematicians in order to get new ideas to tackle the problems in this area. The project is of national scope and it can be best done in Ottawa since access on a daily basis to the Statistics Canada surveys and staff is desirable. However, the required research cannot be carried out effectively within the present framework of Statistics Canada. It needs an independent unit with its own facilities and staff, some of whom would be seconded from existing research groups within Statistics Canada."

4) To coordinate and pursue the study of Systems Analysis applied to Canadian problems. It might serve as the agency through which Canada could relate in a meaningful way to the International Institute for Applied Systems Analysis[6] (IIASA) and to the International Federation of Institutes for Advanced Study[7] (IFIAS). The Futures Institute, proposed by Senator Lamontagne, if it were to come into being, would be largely an exercise in futility unless it had at its core a highly competent mathematical-modelling group.

5) To serve as a halfway house where personnel from universities, community colleges, government and industry would meet and interact intensively for periods - from, say, two to twenty months. The professors would return with a better feeling of the problems which many of their graduates will actually confront. The others would

return with some of their problems solved and with new resources to attack outstanding problems. All would form new contacts which, over the years, might issue in creative collaboration. This was the experience of the Oxford Study Group[3]. A contribution would be made to creating some of the "redundant channels of communication" which Stafford Beer rightly regards as essential if our increasingly complex and interdependent society is to survive.

6) To mount specialized training courses for public servants at all levels of government whenever it becomes apparent that particular topics would be useful to sufficient numbers of people. Even if such courses were not offered by the Institute as such, the officers of the Institute might incite an appropriate university to offer such courses.

7) To provide attractive job opportunities for able young Canadian PhD's, the best of whom might otherwise be drawn away from Canada during the present period of stringent financial conditions in the universities.

8) To act as a central clearing-house for problems in applied mathematics that arise in Canadian governments (federal, provincial, or municipal), industry, business and science. By means of a well run clearing-house, it is argued, researchers in the mathematical sciences would be kept aware of significant unsolved problems and the users would be given help in finding solutions to immediate and long-term problems.

Possible Forms

To accomplish all of the preceding functions will require a very large organization in the long-run. However, strong objections have been raised against the creation of a monolithic institute with a large permanent staff. There is no way known to man of avoiding Newton's famous Fourth Law: All bureaucracies have as their chief aim their own preservation! When this is combined with Newton's First Law, it follows (by mathematical logic) that once created, a monolithic institute would continue forever in an undeviating straight line propelled by its own inertia. Furthermore, the evils of centralization in a country like Canada with extreme regional differences are universally admitted. Undoubtedly, the most distinguished mathematical Institute in the world is the Steklov Institute in the U.S.S.R. which has two branches - in Moscow, and Leningrad.

Any attempt to engage the federal government to provide large funding for a Mathematics Institute should recognize that it has already begun in a small way in its support of the Centre de Recherches Mathematiques in the University of Montreal and that any additional effort should be regarded as a development from that Centre.

The form of the Institute would, to a large extent, be dependent upon the functions which it is created to fulfill. We describe three possible structural patterns. The first two have successful Canadian precedents. However, the third is probably more appropriate in the Canadian circumstances.

(i) The Institute could be a new Division of the National Research Council, parallel to existing Divisions such as Physics or Mechanical Engineering. This is a form of organization which is well understood in Canada. It would mean that the Mathematics Institute could start off very quickly embedded in a significant nexus of first-rate scientific workers who are already attempting to reach out to satisfy the practical and developmental needs in Canada. It would have the disadvantage that the Institute might too easily settle into traditional administrative structures. Moreover, it would be competing with other well-established Divisions of the N.R.C. for the meagre funds which the government has been giving to the Council in recent years and it could be caught in the ethos of the N.R.C. which is dominated by engineers and experimental scientists who might not show great sympathy for modern developments and applications of mathematics.

(ii) The Institute could be located at a university as are certain institutes concerned with transportation or resource development. This would ensure a continuing infusion of new ideas from the faculty of the university, provide for financial accountability, and make possible the offering of short courses through the department of continuing education of the university. It could also contribute to the strengthening and out-reach of the mathematics department of the university in question. If this were the structure decided upon, it would be natural to strengthen and enlarge the Centre des Recherches in the University of Montreal. However, it would not then be possible to satisfy the desiderata of our correspondent who argued that the Institute should play a major role in improving the methodology of surveys in Canada. It is also unlikely that industry and government would finance an Institute tied to one university at anything like the level which will be needed if a truly significant impact is to be made on the problems enumerated in the preceding section.

(iii) An Institute could be created along the lines of the Steklov Institute, with a strong central headquarters and several branches distributed across the country. Each branch could specialize in one or more aspect of the general aims. The subject of specialization could be related to regional problems. For example, a branch in Halifax might concentrate on mathematical modelling for oceanography and the Bay of Fundy tides; one in British Columbia on ecology and forest products; one in the Prairies on engineering and oil exploration; one in Central Canada on computing, financial institutions, and municipal government. The headquarters, in Ottawa, would act as a clearing-house, maintain contact with foreign institutes and pursue, for example, the critical study of survey methodology. Branches could be located in or near universities or in the provincial research foundations. There could be a separate branch specializing in engineering problems and sophisticated computer algorithms as a section of the N.R.C.

Whatever its structure, the Institute should be financed by long-term commitments from government and industry. The Treasury Board and the Ministry of State for Science and Technology could support the activities related to IIASA and the modelling of issues in public policy.[6] Since energy is an issue of grave long-range consequences, the Department of Energy, Mines and Resources would also be keenly interested in mathematical analysis related to energy policy. The Department of Transport, the transportation industry and the Department of Communications would support research pertinent to mathematical problems bearing on transportation and communications. The N.R.C. would be involved with the mathematics of engineering, physics and chemistry.

We would argue that it would not be healthy for the Institute to be funded by one grant from a central Granting Council, for this would allow it to become ingrown and disembodied. Better that its funding be linked directly to industry and departments of the federal government which are expecting some return for their money and in which there are officials who are responsible for continuing liaison - preferably officials with some mathematical training. However, for this concept to be viable, the funding must not be provided by one-year contracts. Some more flexible method involving firm commitments for various periods ranging from three to ten years must be found.

The Institute could explicitly avoid accumulating a large permanent staff by devoting a major portion of its budget to providing the supplementary funds necessary to facilitate the temporary movement of personnel between industry, government, universities and community colleges. (We assume that professors on sabbatical or personnel from industry would continue to receive their normal stipend from the agency from which they came and to which they would be committed to return.)

Such an Institute is unlikely to spring full-blown from the brow of Mr. Drury, but if it were given the go-ahead signal - presumably by the federal government - it would require a gestation period[10] of three to eight years. It would need first-rate personnel from the beginning who combined wide-ranging competence with an imaginative approach to institutional structures.

We therefore recommend,

-that the Ministry of State for Science and Technology advise the government to create a Canadian Institute of Applied Mathematics which would have as one of its principal objectives the promotion of interaction between researchers in the mathematical sciences and the users of mathematics.

Notes - Chapter IX

1. Professor Frank H. Northover of Carleton University, a distinguished Canadian applied mathematicians,is especially noted for his work on diffraction theory.

2. I. Halperin is a Professor at the University of Toronto who is internationally known for his work on operator algebras.

3. Leopold Infeld, who collaborated with Einstein, became a Professor of Mathematical Physics at the University of Toronto. He left in 1950, after being attacked in an irresponsible manner in Parliament by George Drew and returned to his native Poland where he became Director of a first-rate Institute of Theoretical Physics.

4. See Appendix VI. (As we go to press Professor Paul Ribenboin has just returned from six months in France where he gained the impression that the French government has been much more generous and energetic than the Canadian government in its support of pure and applied mathematics.)

5. There is a story about an engineer in one of the government laboratories who requistioned 100 hours of computer time to find the roots of certain expressions involving Bessel functions. An able mathematician, acquainted with G.N. Watson's Theory of Bessel Functions, was able to write a program in a few minutes which solved the problem with 1/2 hour of computer time. The processing of the Mathematics Questionnaire is another case in point. Because the authors consisted of a statistican and two mathematicians, the cost to the Study was reduced very considerably compared with that quoted by a commercial firm or the cost of similar questionnaires to Statistics Canada. A senior official of one of the chartered banks told us, in confidence, that his bank had wasted millions of dollars on computers because they simply could not find competent personnel.

6. IIASA is an Institute and situated in Austria near Vienna supported by 12 countries, including the U.S.S.R. and the U.S.A. It is dedicated to the development of Systems Analysis for peaceful purposes.

7. One correspondent wrote "A Canadian Mathematics Institute could cooperate with the 'International Federation of Institutes for Advanced Study', (IFIAS). This non-profit, non-government organization was founded in 1972 with a secretariat in Stockholm and now has twenty member Institutes. It is undertaking coordinated trans-nation and trans-disciplinary research and analysis likely to make significant impact on global problems. There is a place for applied mathematics in these studies and a Canadian Institute would be welcomed as a partner in the important work which lies ahead for this international organization."

8. On page 354 of Mathematics in Today's World (the report of the Ottawa Seminar), Professor J.H. Blackwell, of the Department of Applied Mathematics of the University of Western Ontario, describes a very successful informal conference, which has been held for several years at Oxford University, between Oxford mathematicians and engineers from industry with specific technical problems.

9. This view has been put forward by several persons including Professor F.M. Arscott, Head of the Department of Applied Mathematics at the University of Manitoba, in his comment on the proposal of Professor Halperin referred to above.

10. To suggest the order of magnitude of this proposal, if it is to be taken seriously, we are thinking (in 1975 dollars) of the following "ballpark" figures: a basic Treasury Board Grant of two million dollars per annum, increasing to five million in eight years. Once the Institute had a few branches, additional funding in the form of three to ten year contracts could be sought from industry, provincial governments and various departments of the federal government.

Chapter X

Canadian Council for the Mathematical Sciences

"In Union There is Strength." - Anon

At its meeting in May 1974, the Joint Committee responsible for the Mathematics Study passed the following resolution unanimously.

"The sponsoring organizations of the Study should be urged to create a continuing Council to promote the development of the mathematical sciences in Canada and their deployment in the national interest."

The Association of the six sponsoring organizations in the Study has opened up the possibility of interaction among groups interested in mathematics in Canada. This interaction holds good promise for the future. The channels of communication which have been brought into being should not be lightly abandoned. Furthermore, it will take several years of concerted effort to ensure that the recommendations of this Study are considered seriously and acted upon in an effective manner. The chief role of the proposed Council will be to guide and maintain such a cooperative effort.

Certain of the original aims of the Study have not been accomplished because of lack of time and resources. Of prime importance are questions related to manpower, an area of which we have, at most, barely scratched the surface.

There is universal lip-service to the proposition that our society is totally dependent for its well-being on technically trained personnel. However, no one claims that even the problem of making a sensible inventory of highly qualified personnel has been solved. We are much further from being able to foresee the number of persons of a particular competence which will be needed in the future, and from formulating a policy for their production and training. An important, though formidable task of the Council, would be to devise a method of monitoring supply, demand and need for mathematically trained persons in Canada. This would supplement the more general third function proposed for the Applied Mathematics Institute.

Other possible purposes of the Council are contained in the suggestions at the end of Chapter III.

The Seminars which were organized in connection with the Study to bring together mathematicians from the universities or community colleges on the one hand, and users of mathematics in industry, business or government on the other, were hailed by many of the participants as highly worthwhile. There is no doubt that a multitude of similar seminars could be conducted to good effect throughout the country. This might be a project sponsored by the Council.

Another might be the development of a slightly different type of seminar aimed at middle and top management, since it was frequently alleged that a timid or ill-informed attitude towards mathematics on the part of management is an important factor inhibiting the effective deployment of mathematics and mathematically trained personnel in our society.

The Council might circulate a list of groups or individuals in universities, industry and government which would welcome mathematical interaction as well as information concerning the interests and problems of the various groups.

The Council might suggest to members of the participating organizations ways in which they could take constructive initiatives to improve the teaching of mathematics in elementary and secondary schools.

The Council could hold a watching brief concerning standards of mathematics at all levels of the educational system and seek to incite a general improvement.

Until a Mathematics Institute is created along the lines described in Chapter IX, the Council might undertake to create a clearing-house to put users of mathematics in touch with mathematicians who are willing to do consulting work,and to advise universities when persons from industry and government would be available as guest lecturers.

Membership in such a Council would not necessarily be restricted to the original six sponsors. Its Constitution should make provision for the adherence of other groups concerned about mathematics. In particular, if one or more organizations emerges which effectively represent mathematics teachers in secondary schools or community colleges, it would be essential to gain their support for the Council.

In order to make even a modest beginning with some of the proposed tasks, financial resources in addition to those which the sponsors of the Study could provide, must be found. Some of the tasks of the Council might be of interest to the Ministry of State for Science and Technology or the Department of Industry, Trade and Commerce, or Manpower and Immigration, or Statistics Canada. An effective Council might become viable if supported by long-term contracts from various departments of the federal government. Furthermore, an effort should be undertaken to obtain contracts with industry and provincial governments in order that the Council be viewed as truly national in its scope.

Chapter XI

An End and a Beginning

"Come, my friends,
'Tis not too late to seek a newer world.
Push off, and sitting well in order smite
The sounding furrows." Tennyson[1]

In Chapter I we recorded the original formulation of the objectives of the Study and also reported that these objectives were only partially attained. In this last chapter, we attempt to evaluate briefly our success in this respect, recapitulate the chief recommendations and point the way forward.

The Objectives Revisited

Let us look at the objectives as stated at the beginning of Chapter I.

The first is partially answered by the results of the Questionnaire and in the Report of the Ottawa Seminars: Mathematics in Today's World[2]. The practial implications of this information are not unambiguous. However, one conclusion is clear. Given the extraordinary importance of applied statistics in our society, it seems that most mathematics departments do not pay enough attention to this subject.

The second objective called for the description and evaluation of research in the mathematical sciences. A mere inventory could have been undertaken but was not deemed to be worth the effort. However, Appendix VIII exemplifies the sort of information bearing on this objective which is readily available. Possibly, it would not be unreasonable to take the Number of Applicants under the several categories of Appendix VIII as a rough indication of the amount of research activity in Canada in different areas of the mathematical sciences. Presumably the ratio of the Number of Awards to the Number of Applicants is a rough index of the quality of that research - as evaluated by the peer judgement system. Of course, it is frequently urged that research should be evaluated in terms of the "national interest" or its practical "usefulness". Abstractly considered, one can argue with some cogency along these lines. However, no one has yet suggested a better system than that which depends on peer judgements. Some, but by no means all, of the pros and cons of this issue were set forth in Chapter VII.

In Chapter I we noted that the problem of estimating future needs for highly qualified personnel is far from solved for any discipline. Certainly, we make no pretense that we have met satisfactorily the third objective. However, we have accumulated much evidence that there has been a considerable need in the past for mathematically trained people and that this need will probably increase rapidly in the immediate future. We are grateful to Otto Tomasek and his colleagues in Bell Canada for providing the Job Forecast reported in Appendix IX. By improving the data base, the method by which that Forecast was obtained could probably be used to provide reasonably dependable forecasts for 2 or 3 years ahead, or possibly even for 4 years. However, the delay time in educating a Master's or Doctoral candidate is 5 to 8 years, so the value of such forecasts is limited. The problem of developing methods of obtaining forecasts of manpower supply and needs is closely related to the question of improving Survey Methodology raised in Chapter IX. This is a key problem in any rational approach to Public Policy. It should have high priority for any National Institute of Applied Mathematics funded by the federal government.

The fourth objective dealt with education in the mathematical sciences. This was the subject which evoked the greatest response to the Study. It is treated in Chapters IV, V and VI.

The last of the original explicit objectives requires us to suggest methods of identifying significant real problem areas amenable to mathematical treatment. That the number of such areas is almost unlimited is suggested in Chapter III and illustrated concretely in <u>Mathematics in Today's World</u>[2]. When unsolved mathematical problems which are crucial for the understanding of some issues of public import are identified, they could be publicized by the proposed Institute for Applied Mathematics or the Canadian Council for the Mathematical Sciences.

Formal Recommendations

The diligent reader of the Report will have noted that scattered through it are a large number of suggestions for action. Nearly all of these were proposed by more than one correspondent to the Study. None of them are advanced dogmatically as ultimate truth. It is our hope that they will all be considered seriously and that such consideration will frequently lead to fruitful action.

However, there are seven recommendations which the Joint Committee of the Mathematics Study regard as of central importance and on which they urge that action be taken as soon as possible.

1. Explicit efforts must be made at many levels of our society - including universities, business, industry, departments of Federal and Provincial Governments and many Professional organizations in order to promote more effective deployment of mathematics and mathematically trained personnel.

This recommendation is intended to sum up all the suggestions of Chapter III.

2. The provincial Ministers of Education should take vigorous action to improve the teaching of mathematics in Elementary and Secondary Schools.

Although directed in the first instance to the attention of the Ministers, the implementation of this recommendation will require active support of students, parents, researchers, professors, mathematical practitioners and Boards of Education over many years.

3. On the initiative of the Canadian Mathematical Congress a task force should be appointed jointly by the six sponsoring organizations to study the programs in the mathematical sciences offered in Canada to Undergraduates, Master's and Doctoral students and to recommend the structure and content of such programs which it deems appropriate in Canada at present and for the immediate future.

4. The Ministry of State for Science and Technology should institute a review of granting policies of the federal government for the support of research to determine (a) how the mathematical community can be more adequately represented at the senior levels of decision-making for federal support of research, (b) how the total financial support, by the federal government, of research in the mathematical sciences can be sustained at an adequate level, (c) whether it

would contribute more effectively to the development of the mathematical sciences in Canada if the current practice of the NRC of awarding most grants to individual researchers, were gradually modified to emphasize somewhat more strongly the support of projects which bring Canadian researchers in the mathematical sciences into active interaction.

5. The officers of Canadian Universities should provide increased incentives for academics in the mathematical sciences to make greater efforts to communicate on a professional basis with the users of mathematics.

6. The Ministry of State for Science and Technology should advise the federal government to create a Canadian Institute of Applied Mathematics which would have as one of its principal objectives the promotion of interaction between mathematical researchers and the users of mathematics.

7. The sponsoring organizations of the Study should create a continuing Council to promote the development of the Mathematical Sciences in Canada and their deployment in the national interest.

A Newer World

In 1975, when these lines are being written, anyone who is sensitive to the global social process must regard the future with considerable apprehension. We confront an energy crisis, rising unemployment, galloping inflation, the degradation of our environment, an increase in violent crime, an explosion of the world population and the imminent exhaustion of several vital non-renewable resources. Is this an epoch when we dare "waste" time worrying about the role of mathematics in Canada?

Despite the many frightening possibilities which beset us, there has never been an epoch in human history when the options available to humanity were more exciting or more pregnant with possibilities for good. But if we are to seize these opportunities for a fuller life for all people, we shall need to bring to bear upon our problems our utmost powers of rationality - we shall need to mathematize. Of course, we shall need to do other things - to discipline our greed and self-interest, to understand sympathetically the ambitions of other peoples, to employ creatively the physical, engineering, biological and social sciences, to act boldly and decisively ... and much more.

Perhaps mathematizing will be only [3] 10 percent of our effort, perhaps only 3 percent, perhaps only 1 percent - but it will be essential. Apart from the knowledge and control of abstract structure which only mathematics can provide, our contemporary complex technological society would dissolve in chaos.

It is, therefore, urgent that we address ourselves to the tasks which this Report has attempted - however inadequately - to delineate. As we go forward together, we shall learn what is important and what is unimportant. Hopefully, we shall gain new insights and new courage.

Come, my friends. Perhaps it is not too late.

Notes - Chapter XI

1. From the end of Tennyson's *Ulysses*.

2. Described in the section on the Eight Seminars in Chapter I.

3. Recall the comment of W.E. Krause quoted in the section on consciousness-raising in Chapter III.

APPENDICES

APPENDIX I

Results of the Mathematics Survey

We talked about the Mathematical Ecosystem in the second chapter and described briefly its five components and some of its linkages. We also discussed the mechanisms employed to gather information regarding this ecosystem. Briefs were called for; eight one-day seminars were held; Universities, Industry and Government Agencies were visited; and a questionnaire (Table 1) was circulated widely.

The findings of the survey questionnaire were used by the study leaders in concert with the information gathered through the other three mechanisms. No one mechanism was capable of covering the many different points of view regarding the mathematical ecosystem. A valid picture was felt possible only through a multifaceted approach such as the one employed by this study.

The survey was designed to focus on the movement of people from University programs in the Mathematical Sciences into working the world. We wanted to discover what kind of jobs mathematics graduates have been obtaining; what they have been making of their mathematical education; and what opinions they now have concerning the adequacy of this education in preparing them for careers.

The methodology and questionnaire used to obtain this information are described below along with a discussion of the potential bias introduced as a consequence of the sizeable non-response group as well as the resulting statistical inferences one is able to make.

Survey Methodology

In the Fall of 1973, the study leaders wrote to each mathematics department in every Canadian University requesting a complete mailing list of all their graduates

(i) at the bachelor's and master's levels, in the years 1960, 1965, 1970, 1971, 1972 and 1973;

(ii) at the PhD level, in every year from 1960 to 1973 inclusive.

Most of the mathematics departments had never attempted to contact their graduates before, and so a great deal of effort was involved in compiling the mailing lists. Special difficulties were encountered in dealing with students who did not taken an honours degree. It was finally decided to request Departments to include about 10% of these graduates in general programs who had taken 5 or more mathematics courses. Despite these and other difficulties, most university mathematics departments were extremely cooperative, and agreed to mail out questionnaires which had been designed and tested by the study group.

The questionnaires, accompanied by stamped self-return envelopes, were returned directly to the study centre at the Science Council of Canada. Mr. K. Beltzner, bound by oath under the Official Secrecy Act, was called upon to check-in the questionnaires (through the attached code numbers) and to remove the top page containing the name and address of the respondent from the rest of the questionnaire.

The top pages were subsequently used in the preparation of a mailing list of recipients for the Mathematics report.

The processing of the questionnaire was at first contracted-out under authority of the Joint Committee, but reverted back to the study group when it became evident after a series of long delays that the contractual terms could not be honoured by the firm.

Under the direction of the study leaders, the questionnaire was processed in accordance with the following steps: (All computer processing was done at Queen's University at Kingston.)

Step 1: The questionnaire was scanned for completeness and accuracy. An extensive pre-coding operation followed, coding the names of educational institutions, province or country of citizenship, province or country where the respondent completed his/her high school education, and province or country of employment.

Step 2: The pre-coded questionnaire was then transcribed onto specially designed coding sheets, followed by a fully-verified key punch operation.

Step 3: The data cards were loaded onto computer tape using a standard utility program. The raw data were then subjected to an extensive edit written by the study group and consisting of hundreds of consistency and cross checks. All errors detected by the computer edit were analyzed and after correction, re-introduced to the edit system.

Step 4: The clean data tape was used in the construction of an SPSS[2] file. Numerous sub-files, including files of all bachelor, master and doctoral graduates from Canadian Universities[1] in the selected years were costructed. Sub-files of members of the sponsoring societies were also constructed and kept on computer tape files for future research purposes.

Step 5: Various sub-files were used in the tabulation and analysis of the questionnaire.

Non-response bias and statistical inference

In every survey of this nature, there is always a problem in knowing what to do about the non-respondents. Why do some people receive, fill in, and return questionnaires while others do not? If it were simply a matter of chance, one would expect the information gleaned from the respondents to be typical of non-respondents as well. But the two groups are most probably not similar. It may be that the people who answered the questionnaire are the very ones who have the most complaints to make; or who are the most vocal about their complaints. Non-respondents certainly include people who have moved around a great deal or who have changed their names through marriage, and have therefore not received a questionnaire. (In fact, this is the single largest cause of this survey's low response rate (see Table 2). Some people who did receive the questionnaire may have refused to answer it for reasons which stem from their own personal attitudes toward mathematics, toward education, or toward questionnaires in general, or toward this instrument in particular.

It must be borne in mind, therefore, that the data and the opinions which are reported herein do not necessarily respresent the entire population of mathematics graduates in Canada, but only those who responded to the questionnaire (more than eighteen hundred individuals in all). Any attempt to draw statistical inferences about the total population of mathematics graduates in Canada on the basis of our survey results alone is unwarranted, and we would strongly resist any efforts to do so.

Indeed, for the purpose of this study, it was not strictly necessary to obtain a 100% representative sample from a statistical point of view - for exactly the same reasons that it was not necessary to obtain 100% representation for the other mechanisms employed to gather opinions, namely the call for briefs, the conferences held and the visits made!

The fact that the views expressed to the study through the various mechanisms were found not to be in conflict when it came to the identification of major problem areas suggests that little value could have been gained by spending significantly more resources ensuring that an absolutely representative statistical sample was at hand. Furthermore, the cross-checks available through the various mechanisms were felt to result in a much more accurate picture of the teaching at about 34%. This leaves the majority employed in government and industry.

Table A - Academic Employment of Mathematical Sciences Graduates by Degree Level

	Mathematics Survey		HQM[1] Survey
	Working in Educational Institutions	Teaching as primary nature of work	Occupation: Teaching
Degree Level			
B.A.	29%	22%	27%
M.A.	45%	37%	41%
PhD	82%	78%	35%
Total	37%	30%	34%

[1] Highly Qualified Manpower Post-Censal (1973) Survey.

Notes: 1. The primary survey recorded both category of employer and nature of work, the HQM survey recorded occupation.

2. Full time students working on a higher degree are included in both surveys.

Source: Primary research by the authors, (Table 1.I), HQM Survey.

Highlights of the Math Questionnaire

> *"One of the main objectives of the Mathematics Study is to find out what kind of career opportunities there are for young men and women with mathematical capabilities, and to feed this information back to university professors. We have excellent*

programs for developing academics - people who intend to be teachers at the high school of university level - but we need a new breed of cat. Perhaps 80% of our graduates in future will be going out into the world of government and industry. Not too many professors know what tools they will need or what problems they will encounter."

- Comment by Professor C.F.A. Beaumont, Associate Dean of the Faculty of Mathematics, University of Waterloo, at the Ottawa Seminar.

By and large Professor Beaumont's future is not too far distant. Our primary survey research and the Ministry of State for Science and Technology's "Highly Qualified Manpower Survey" both place the percentage of math grads who's primary nature of work or occupation is

The same picture also held true for the chemist and physics B.A. Hon. Grads, the subject of an earlier highly focussed study by Drs. A.D. Boyd and A.C. Gross [3] and published as a background study for the Science Council of Canada. These authors concluded that the honours program (in chemistry and physics) is in need of reassessment, and that educators must perform this task in the near future.

Table 1.I provides detailed tabulations by degree level of the employment patterns by industry and primary function of the respondents to our primary survey. Similar tabulations showing the results of the HQM Survey will hopefully be available in time for our final edition.

Mathematics Education

The respondents were asked to reflect upon their mathematical education and express opinions regarding a variety of questions dealing with the emphasis that was placed on the totality of their mathematics education. Most questions were answerable through a rating scheme of whether there was sufficient, too much, or too little emphasis given. The results of these enquiries are presented below and are broken out separately by degree level.

(a) Mathematics Bachelor Graduates

The responses to Question II.G of the mathematics questionnaire are tabulated in Tables II.G.1 - 3. These tabulations shed considerable light on the opinions held by the B.A. group regarding the emphasis that was placed on the different mathematics subject areas and the importance of these subject areas in their work.

Table II.G.1 reports on the mathematical education received by these graduates at the high school level. Here, Algebra, Geometry and Trigonometry were seen by the majority as being given sufficient attention. However, Set Theory, a relatively new subject area, was given a mixed rating.

Tables II.G.1 and 2 reveal a high degree of satisfaction with the emphasis that was placed on Analysis (Calculus and Differential Equations), and the importance of Computer Science (Computer Programming, Simulation Techniques, Systems Analysis) as well as the lack of importance of Topology (Point Set, Algebraic, Manifold) and University Geometry (Solid, Differential, Projective) in their work. Also, of interest was the overall consistency by year in the direction away from the "About Right" option. Except for Calculus, more respondents repeatedly felt that the emphasis which was placed on the different mathematical subject areas was "too little" rather than "too much".

It must be emphasized that we did not ask "why" the respondent felt that any subject area did not receive sufficient emphasis. The reasons undoubtedly are numerous and varied. For example, one respondent to our questionnaire spoke out at the 1974 International Mathematics Congress Meeting in Vancouver and said that the reason for his stating that computer science received insufficient attention was that when he went to university, computers were not yet widely used.

However, for whatever reasons, the fact that the majority in all graduating classes found these subject areas as receiving insufficient emphasis points toward the need of reassessing the curriculum of the math "streams" - especially since a number of these subject areas are the ones identified as being of greatest importance in the graduates[4] present job.

Questions were also asked regarding the emphasis placed on the various mechanisms employed to "teach" mathematics to the undergraduates, (Table III.C.1.1 and 1.2). The majority felt that the easy[3] mechanisms: written assignments, lectures, independent study, and examinations received sufficient if not too much attention. On the other hand, the majority felt that original work, seminars, group discussions, and teamwork were lacking.

Turning to the type of material taught in the universities, the picture is no different. (Tables III.D.1.1 and 1.2) Theoretical development of mathematics and well formulated mathematical problems were seen as receiving adequate attention whereas insufficient time was spent in stimulating the respondents with useful mathematical results and introducing them to unstructured problems.

Similarly, whereas the respondents felt that sufficient emphasis was placed on the development of computation skills, they felt that insufficient emphasis was placed on teaching the art of communicating mathematical ideas to others, or discussing the interconnections between the various branches of mathematics or even how to use the mathematical literature. Furthermore, when queried about such things as the consideration given to the impact of mathematics on society or the cultural aspects of mathematics, the overwhelming response was that "too little" was given.

(b) Mathematics Masters and Doctoral Graduates

Generally, the master and doctoral graduates felt more satisfied with their mathematics education that the bachelor graduates.

The differences in the emphasis placed on the various mechanisms employed to "teach" mathematics to the undergraduate and post-graduate students is evident in their opinions as reported in Tables III. C.2 and 3.

For the master graduates, their mathematics graduate education provided sufficient emphasis on unstructured problems and useful results, original work and seminars. Over half of this group, however, felt that examinations were emphasized too much.

The doctoral graduates also felt that more emphasis could have been placed on mathematical modelling and applications to other fields as well as to the history, philosophy, and the social implications of mathematics.

The social, cultural, philisophical and historical aspects of mathematics were generally seen as being insufficiently emphasized in the mathematical education of all the respondents to the questionnaire.

Mathematics and Jobs

In order to relate mathematics to jobs, only those respondents employed full-time at the time of the survey are considered. Thus, "students" are not included. Further, we are considering only the graduates of Canadian Universities [1] in certain select years. Those individuals who responded to the questionnaire through the society mailing lists will be, however, included in the final edition of this report.

Question I.G asks the respondents to indicate whether or not they consider their job related to mathematics. We find that most do. In fact, the proportion of respondents who's jobs are related to mathematics increases with the level of the degree held, from 80% for the B.A. group to 98% for the PhD group (Table I.G). The most frequently cited reason for not choosing a mathematics related job was the preference to do something else. Neither the location of nor higher compensation for non-mathematical jobs appear to be significant factors in the decision affecting a graduates' opinion for a job unrelated to mathematics.

Question II.A explores the issue of the necessity of a mathematics degree in the graduate's job as seen through the graduate's own eyes and according to the graduate's employer (Table II.A). Of particular interest is the proportion of respondents, by degree level, who indicated that their job according to their employer, requires a degree in the mathematical sciences. This proportion increases from 39.0% for the B.A.'s, to 61.7% for the M.A.'s, to 84.1% for the PhD's.

Noteworthy, although not statistically significant, is the finding that these proportions are consistently higher in the respondents own opinion. That is, there appears to be some slight evidence that employers do not fully appreciate the extent to which a degree in mathematics is required in some jobs. This evidence, however, is not nearly as convincing as the strong opinions expressed by managers in our industrial seminars. In every seminar some managers presented the attitude that whereas they felt that mathematics is very important in their organization, mathematicians are not.

Question II.B of the questionnaire asks the respondents to display their approximate distribution of total working time among different functions and to list their first three principal activities in the order of their relative importance. Table II.B.1 displays these first, second and third principal activities by degree level.

As expected, in light of Table I.1 the principal activity of the PhD group is teaching mathematics and conducting mathematical research. For the M.A. group, investigating unstructured problems, increasing specialized knowledge through reading, attending seminars, etc., as well as teaching mathematics at the college level are cited most frequently as the principal activity. The B.A. group's principal activities are applying routine mathematical procedures, investigating unstructured problems and "other" functions.

The percent distribution of the respondent's total working time by degree level is summarized in Table II.B.2.

Job satisfaction which included the utilization of training was probed in Question II.F. Numerical answers were sought to a variety of characteristics and were recorded on a rating scale which ranged from "hardly at all" (20-40) to "to a high degree" (61-80). Detailed tables, Table II.F.1 - 4 are provided for the reader.

The results to Question II.F. indicate that most graduates were satisfied with their work environment. Yet, despite the strong "central tendency", differences are observable on various questions. To better visualize and discriminate these differences, the reader is referred to the more detailed tables, which group the responses into six discrete groups.

Utilization of training is seen as a particularly significant concept by educational planners and is worth commenting on separately. Whereas one speaks readily of the Ph.D. being underutilized, the question of whether or not the B.A. or M.A. is underutilized is not given much attention. This is primarily due to the significantly lower cost associated with educating a B.A. or M.A. However, the Tables II.F.1 - 4 illustrate that as far as the respondents to our questionnaire are concerned, 33.8% of the B.A. group and 31.6% of the M.A. group feel "underutilized" with respect to their mathematical training. The "underutilization" of the Ph.D. is 11.5%.

The selected highlights presented above represent only a small fraction of the information available as a result of the primary survey. More focussed results will be forthcoming periodically and comparisons withthe results of Special Study No. 28, "Education and Jobs" by A.D. Boyd and A.C. Gross, and with the results of the Highly Qualified Manpower Post-Censal (1973) Survey are also planned.

Notes - Appendix I

1. The questionnaires were sent out by the following Canadian University Departments:
ACADIA UNIVERSITY - Computer Science
UNIVERSITY OF Alberta - Mathematics
ALTHOUSE COLLEGE OF EDUCATION (U. of Western Ontario) - Mathematics (Education)
BISHOP'S UNIVERSITY - Mathematics
UNIVERSITY OF BRITISH COLUMBIA - Computer Science
BROCK UNIVERSITY - Mathematics
UNIVERSITY OF CALGARY - Mathematics
DALHOUSIE UNIVERSITY - Mathematics
UNIVERSITY OF GUELPH - Mathematics and Statistics
LAKEHEAD UNIVERSITY - Mathematics
UNIVERSITE LAVAL - Mathematics, Computer Science
UNIVERSITY OF LETHBRIDGE - Mathematical Sciences
LOYOLA COLLEGE - Computer Science
UNIVERSITY OF MANITOBA - Mathematics, Statistics
UNIVERSITY OF McGILL - Mathematics
MEMORIAL UNIVERSITY OF NEWFOUNDLAND - Mathematics
UNIVERSITE DE MONCTON - Mathematics
UNIVERSITY OF NEW BRUNSWICK - Mathematics
OTTAWA UNIVERSITY - Mathematics
UNIVERSITY OF PRINCE EDWARD ISLAND - Mathematics
UNIVERSITE DU QUEBEC a Chicoutimi - Mathematics
a Montreal - Mathematics
a Trois Rivieres - Mathematics
QUEEN'S UNIVERSITY - Mathematics, Computing and Information Science
ROYAL MILITARY COLLEGE OF CANADA - Mathematics
UNIVERSITY OF SASKATCHEWAN - (Regina) - Mathematics
(Saskatoon) - Mathematics, Computer Science
UNIVERSITE DE SHERBROOKE - Mathematics
SIMON FRASER UNIVERSITY - Mathematics
SIR GEORGE WILLIAMS UNIVERSITY - Mathematics
UNIVERSITY OF TORONTO - Mathematics, Computer Science
UNIVERSITY OF VICTORIA - Mathematics
UNIVERSITY OF WATERLOO - Faculty of Mathematics
UNIVERSITY OF WESTERN ONTARIO - Mathematics, Applied Mathematics and Computer Science
UNIVERSITY OF WINDSOR - Mathematics and Computer Science
UNIVERSITY OF WINNIPEG - Mathematics and Statistics
YORK UNIVERSITY - Mathematics

2. SPSS, or Statistical Package for the Social Sciences, is a 'software' package (or set of computer programs available at most University computer installations.

3. Special Study No. 28, - "Education and Jobs", p. 67.

4. See Chapter VI.

MATHEMATICS IN CANADA

a study sponsored by

Canadian Mathematical Congress
Canadian Institute of Actuaries
Canadian Operational Research Society
Canadian Information Processing Society
Statistical Science Association of Canada
American Statistical Association (District 11)

c/o Science Council of Canada
Conseil des Sciences du Canada
150 Kent Street
7th Floor
Ottawa, Ontario
K1P 5P4

Tel. (613) 995-6853

November 1973

THE MATHEMATICS QUESTIONNAIRE

1. DO NOT COMPLETE THIS QUESTIONNAIRE MORE THAN ONCE. If you have already completed and returned the questionnaire, please accept our thanks and destroy this copy.

2. THE ANSWERS WHICH YOU GIVE WILL BE KEPT IN THE STRICTEST CONFIDENCE. Each university will have access to information concerning its own graduates, but no data regarding individuals or organizations will be published.

3. WHAT ARE THE MATHEMATICAL SCIENCES? For the *purposes of this questionnaire,* the Mathematical Sciences consist of Pure Mathematics, Applied Mathematics, Mathematical Statistics, Actuarial Science, Operations Research, and Computer Science. The words "mathematics", "mathematical", and "mathematician" refer to any or all of these fields.

4. FOR YOUR CONVENIENCE IN ANSWERING: All spaces for answers appear as white areas on a blue background. Almost all of the questions can be answered with a checkmark or a number. If a question is not applicable to your present situation, skip it. Comments on selected questions appear on page 1.

5. HOW TO ANSWER THE OPINION QUESTIONS: Some of the questions in sections II and III of the questionnaire ask for your opinion on the strengths and weaknesses of your mathematical education. In each case, we are seeking your considered opinion, based on *your* subsequent experience and *your* views of the purpose of education.

6. ADDITIONAL INPUT TO THE STUDY: You are invited to give fuller expression to your views on the role of mathematics in our society and its relation to your own career, by sending a letter or a brief to the Mathematics Study, c/o The Science Council of Canada, at the address given above. Aim for January 15, 1974, as a deadline.

7. SUMMARY OF RESULTS: If you wish to receive a summary of the results of this survey, please give your name and address in the space provided below. THIS IS ENTIRELY OPTIONAL. YOUR NAME WILL NOT BE ASSOCIATED WITH YOUR COMPLETED QUESTIONNAIRE IN ANY WAY. IF YOU PREFER, YOU MAY DETACH THIS SHEET AND MAIL IT SEPARATELY.

NAME (Print) ______________________________

ADDRESS ______________________________

____

____

COMMENTS ON SELECTED QUESTIONS

I. BIOGRAPHICAL AND PROFESSIONAL INFORMATION

A. Please use more than one code number to describe joint specialties or special fields; e.g. hydrodynamics (5 & 6) or econometrics (1 & 8).

C. This question will give some information on migration patterns.

D. Half courses are used to avoid fractions. Tutorials and problem sessions are not included in the three hours per week.

E. The sponsoring societies are identified on the letterhead. The other societies are: (g) American Mathematical Society; (h) Association for Computing Machinery; (i) The Institute of Management Sciences; (j) The Society for Industrial and Applied Mathematics; (k) The Mathematical Association of America. You may attach a list of other mathematical societies to which you belong, if you wish.

II. MATHEMATICS AND YOUR PRESENT JOB

B. This question should be answered in PENCIL first, to allow for easy changes. Please check to see that your percentages total 100.

C. The answer you give will be used to convert the percentages of question B into man-hour units, for purposes of aggregation.

G. Any subject areas which are not relevant to your education or your work may be left blank; they will be coded as "4".

H. In this context, "unpublished papers" refers to papers which have been prepared and used for some internal purpose; the phrase does not refer to papers which have been written but never published or used for any purpose.

III. REFLECTIONS ON YOUR MATHEMATICAL EDUCATION

A. In expressing your opinion, please take into account the conditions that existed at the time of your undergraduate education.

I. BIOGRAPHICAL AND PROFESSIONAL INFORMATION

A. Please cite all post-secondary *DEGREES* or *DIPLOMAS* (if *honours* degree, please indicate). From the list given below, select one, two, or three phrases to describe your *FIELD(S) OF CONCENTRATION,* and enter the appropriate *code numbers* in the table.

Degree or Diploma –*honours?*	Year awarded or expected *e.g. '65, '74.*	Fields of Concentration *see code*			Name of Institution	Province/Country	FOR OFFICE USE ONLY

FIELDS OF CONCENTRATION

1. Mathematical Statistics
2. Actuarial Science
3. Operations Research
4. Computer Science
5. Mathematics (other)
6. Physical & Earth Sciences
7. Biological & Medical Sciences
8. Social & Behavioural Sciences
9. Business & Commerce
10. Humanities & Languages
11. Engineering
12. Education
13. Law
14. Other Professional Fields

B. a) Age . . . . ______

b) Sex . . . . ______

c) Citizenship . . . . . . ____________________

C. In what *PROVINCE* or *COUNTRY* did you complete your *HIGH SCHOOL* education?

____

D. Approximately how many *HALF COURSES* did you complete at the *COLLEGE* or *UNDERGRADUATE UNIVERSITY* level in the following areas of Mathematical Science (before graduation)?

a) Pure Mathematics . . . . ______
b) Probability & Statistics . . . . ______
c) Computer Science . . . . ______
d) Applied Mathematics . . . . ______

e) *TOTAL* . . . . ______

HALF COURSE: 3 hours/week for about 4 months, or equivalent.

APPLIED MATH: includes O.R., actuarial math, econometrics, mathematical physics, etc.

E. Please check (√) the boxes corresponding to the *PROFESSIONAL MATHEMATICAL SOCIETIES* to which you belong.

Sponsoring Societies

a) ☐ C.M.C.
b) ☐ C.I.P.S.
c) ☐ C.O.R.S.
d) ☐ S.S.A.C.
e) ☐ A.S.A.
f) ☐ C.I.A.

Other Societies

g) ☐ A.M.S.
h) ☐ A.C.M.
i) ☐ T.I.M.S.
j) ☐ S.I.A.M.
k) ☐ M.A.A.
l) ☐ Others: ______ (*number please*)

F. What is your current *EMPLOYMENT STATUS?*

a) ☐ employed full time
b) ☐ employed part time
c) ☐ on leave of absence
d) ☐ self-employed
e) ☐ housewife
f) ☐ student
g) ☐ retired
h) ☐ other (specify) ____________________

G. Is your present job *RELATED TO MATHEMATICS?*

a) ☐ YES b) ☐ NO

If not, what was your *MOST IMPORTANT REASON* for choosing a non-mathematical job?

a) ☐ preferred to do something other than math
b) ☐ promoted out of mathematics position
c) ☐ pay is better
d) ☐ locational preference
e) ☐ mathematical position not available
f) ☐ other (specify) ____________________

H. Have you ever accepted a *POSTDOCTORAL FELLOWSHIP* or similar appointment?

a) ☐ YES b) ☐ NO

If you have, what was your *MOST IMPORTANT REASON* for accepting the appointment?

a) ☐ sought additional research experience
b) ☐ hoped for permanent job as a result
c) ☐ alternative employment not *available*
d) ☐ alternative employment not *desirable*
e) ☐ opportunity to change fields
f) ☐ opportunity to travel
g) ☐ other (specify) ____________________

OFFICE USE ONLY: B. |__|__|__| C. |__|__|__|

I. Please provide information on each job held since completing your undergraduate education. Start with your present job and work backwards in time. Promotions should be listed as *SEPARATE JOBS* whenever they involve a change in job title or nature of work. *Note:* (1) Most of the questions can be answered simply by inserting a number. (2) Use a separate row for each job or promotion. (3) Use a separate row for lengthy sickness, leaves, etc.

OFFICE USE ONLY

1.
2.
3.
4.
5.
6.
7.

DATES OF EMPLOYMENT

Please give beginning month and year, and ending month and year, for each employer.

from to

NAME, LOCATION, and CATEGORY OF EMPLOYER

Give the *name* and *location* of each employer (see examples below). In each case choose ONE number from the following list to indicate the category of your employer.

CATEGORY OF EMPLOYER

Primary Industry	1
Manufacturing	2
Transportation & Communication	3
Public Utilities	4
Trade (Wholesale & Retail)	5
Finance and Insurance	6
Business, Service Industries	7
Military	8
Federal Government	9
Provincial Government	10
Other Government	11
University	12
College (post-secondary)	13
High School	14
Grade School	15
Other (specify in each case)	16

SPECIFIC JOB TITLE and NATURE OF WORK

For each position enter your *official job title.* In addition, choose up to THREE numbers from the list to describe the nature of your work.

NATURE OF WORK

Teaching & Training	1
Basic Mathematical Research	2
Applied Mathematical Research	3
Mathematical Modelling	4
Actuarial & Business Math	5
Operations Research	6
Programming & Dataprocessing	7
Systems Analysis	8
Simulation & Systems Engineering	9
Other Computer Work	10
Sampling & Surveying	11
Experimental Design	12
Other Statistical Procedures	13
Consulting (mathematical)	14
Supervision (related to math)	15
Administration	16
Other (specify in each case)	17

GROSS ANNUAL SALARY BEFORE TAXES

In each case, give *beginning salary & ending salary,* measured in thousands of dollars, & rounded off to the nearest thousand.

beginning ending

PRIMARY OR MAIN REASON FOR LEAVING THIS POSITION

Choose ONE number in each case and enter below (see examples).

promoted	1
exceptional job opportunity	2
to gain more experience	3
study for advanced degree	4
wanted other type of work	5
climate, location	6
physical or mental health	7
inadequate pay	8
little opportunity to advance	9
poor working conditions	10
low recognition of mathematicians	11
big operation: lost in crowd	12
small operation: not enough scope	13
group layoff; fired; contract not renewed	14
family responsibilities	15
other (specify in each case)	16

	Dates: from	Dates: to	Name, Location of Employer	Category	Job Title	Nature	Nature	Nature	Salary beginning	Salary ending	Reason for leaving
Example	April 1956	Aug. 1966	University of Ottawa Ottawa, Ontario	12	Assistant Professor	1	2		7	9	1
Example	June 1972	Sept. 1973	Statistics Canada Ottawa, Ontario	9	Statistician	10	11	15	9	9	2
1. *Present Pos'n*											
2. *Previous Pos'n*											
3.											
4.											
5.											
6.											
7.											

II. MATHEMATICS AND YOUR PRESENT JOB

A. What kind of *ACADEMIC QUALIFICATIONS* are required in your present job? Please check ONE box in each column.

	According to your Employer	In your own Opinion
NO DEGREE REQUIRED:		
a) technical training would suffice	☐	☐
BACHELOR'S DEGREE REQUIRED:		
b) in the Mathematical Sciences	☐	☐
c) not necessarily in the Mathematical Sciences	☐	☐
GRADUATE DEGREE REQUIRED:		
d) in the Mathematical Sciences	☐	☐
e) not necessarily in the Mathematical Sciences	☐	☐

B. Please show the approximate *DISTRIBUTION* of your *TOTAL WORKING TIME* among the different *FUNCTIONS*, to the nearest 5% or 10%. (Include preparation and work done at home.)

a) applying *routine mathematical procedures* . . . . . ____%
b) *supervising* or *evaluating* mathematical work . . . . . ____%
c) *advising* non-mathematicians on math-related problems . . . . . ____%
d) investigating *unstructured problems* . . . . . ____%
e) doing basic or applied mathematical *research* . . . . . ____%
f) *writing* mathematical papers, books, or reports . . . . . ____%
g) increasing specialized knowledge: *reading, seminars, etc* . . . . . ____%
h) teaching mathematics at the *post-graduate* level . . . . . ____%
i) teaching math at the *college* or *undergraduate* level . . . . . ____%
j) other *mathematical teaching* . . . . . ____%
k) *non-mathematical teaching* . . . . . ____%
l) administration . . . . . ____%
m) other functions . . . . . ____%

100 %

Which of the above functions do you regard as your *two or three principal activities*, in your own order of importance?

First: ____; Second: ____; Third: ____;

C. How much *TIME* do you spend on your *PRESENT JOB* (including preparation and work done at home)?

a) ________ hours/week, out of

b) ________ weeks/year.

D. In addition to your full-time work, do you *TEACH* on a part-time or summer basis?

a) ☐ YES: ______course-hours/year
b) ☐ NO.

E. Please indicate how many *MATHEMATICAL JOURNALS* you read regularly. Also, please indicate how many *CONFERENCES* and *NON-DEGREE COURSES* in the Mathematical Sciences you have attended during the past two years.

a) journals read . . . . . ____
b) conferences attended . . . . . ____
c) courses attended . . . . . ____
(*not for degree*)

F. Using the scale and the words, select a number from 20 to 80 to show the extent to which each *CHARACTERISTIC* is present in your job now.

20 — *40* — *60* — *80*

hardly at all — *moderately* — *to a high degree*

a) congenial colleagues . . . . . ____
b) good working conditions . . . . . ____
c) good salary . . . . . ____
d) job security . . . . . ____
e) promotion prospects . . . . . ____
f) novelty & variety of employment . . . . . ____
g) utilization of training . . . . . ____
h) personal satisfaction . . . . . ____
i) scope for individual initiative . . . . . ____
j) intellectual stimulation . . . . . ____
k) prestige inside peer group . . . . . ____
l) prestige outside peer group . . . . . ____
m) social usefulness . . . . . ____

G. Below is a list of mathematical subject areas. Opposite each subject area, insert *TWO CODE NUMBERS* in response to the following two questions:

(a) Looking back on your *FORMAL MATHEMATICAL EDUCATION,* what is your opinion of the emphasis that was placed on each subject area?

CODED ANSWERS

1. Too much
2. About Right
3. Too Little
4. No Opinion

(b) How important is this subject area when you *USE MATHEMATICS IN YOUR WORK?*

CODED ANSWERS

1. Great Importance
2. Moderate Importance
3. Little Importance
4. No Importance

	(a)	(b)
1. *ELEMENTARY MATHEMATICS*		
high school algebra	___	___
high school geometry	___	___
trigonometry	___	___
elementary set theory	___	___
2. *FINITE MATHEMATICS*		
numerical analysis	___	___
combinatorics	___	___
graphs and networks	___	___
other	___	___
3. *UNIVERSITY ALGEBRA*		
number theory	___	___
linear algebra	___	___
boolean algebra	___	___
other	___	___
4. *UNIVERSITY GEOMETRY*		
solid geometry	___	___
diff'l geometry	___	___
projective geometry	___	___
other	___	___
5. *ANALYSIS*		
elementary calculus*	___	___
advanced calculus	___	___
diff'l equations	___	___
other	___	___
6. *TOPOLOGY*		
point set topology	___	___
algebraic topology	___	___
manifold theory	___	___
other	___	___
7. *OPTIMIZATION*		
linear & dynamic programming	___	___
variational principles	___	___
control theory	___	___
other optimization techniques	___	___
8. *COMPUTER SCIENCE*		
computer programming	___	___
simulation techniques	___	___
systems analysis	___	___
other	___	___
9. *PROBABILITY & STATISTICS*		
elementary statistics**	___	___
other statistics	___	___
stochastic processes	___	___
queuing theory	___	___
probability theory	___	___
10. *APPLICATIONS OF MATHEMATICS*		
actuarial science	___	___
operations research	___	___
mathematical physics	___	___
other applied mathematics	___	___
11. *OTHER MATHEMATICS (write in)*		
___	___	___
___	___	___
___	___	___
___	___	___
___	___	___
___	___	___

*elementary calculus includes basic notions and standard applications of ordinary and partial derivatives, Riemann integration sequences and series, and Taylor expansions

**elementary statistics includes basic concepts of statistics and routine procedures such as regression analysis, chi-square tests, and analysis of variance

H. Please give the *APPROXIMATE NUMBERS* of mathematical papers, reports, and books you have written from 1966 to the present. Indicate *SUBJECT AREAS* by (a) consulting the list in question G above, and (b) inserting the corresponding *NUMBERS* in the spaces provided.

Year	*Unpublished Papers*	*Published Papers*	*Books*	*Subject Areas*
1966	___	___	___	___
1967	___	___	___	___
1968	___	___	___	___
1969	___	___	___	___
1970	___	___	___	___
1971	___	___	___	___
1972	___	___	___	___
1973	___	___	___	___

III. REFLECTIONS ON YOUR MATHEMATICAL EDUCATION

A. Please estimate the *PERCENTAGE OF CLASS TIME* devoted to each of the following fields in your *UNDERGRADUATE* education. In each case, you are invited to express an opinion by inserting a code number corresponding to the following:

CODED ANSWERS

1. Too Much 2. About Right
3. Too Little 4. No Opinion

	Class Time	Your Opinion
a) Mathematical Sciences	____ %	____
b) Physical/Earth Sciences	____ %	____
c) Engineering	____ %	____
d) Biological/Medical Sciences	____ %	____
e) Social/Behavioural Sciences	____ %	____
f) Business & Commerce	____ %	____
g) Humanities & Languages	____ %	____
h) Other	____	____
	100 %	

B. What was your *KEY REASON* when you chose to specialize in the Mathematical Sciences? Please check ONE box. If unable to decide, you may check more than one box; your answers will then be given equal weight.

a) ☐ fascination with mathematical sciences
b) ☐ interest in applications to other sciences
c) ☐ interest in "real life" applications
d) ☐ special aptitude in mathematical sciences
e) ☐ attracted by career opportunities
f) ☐ motivated by family, friends, or teachers
g) ☐ more challenging than other fields
h) ☐ less demanding than other fields
i) ☐ other (specify)________________
j) ☐ don't know

C. Please give an opinion regarding the emphasis placed on the following elements of *CONTENT & METHODOLOGY* in your *UNIVERSITY MATHEMATICAL EDUCATION.*

CODED ANSWERS

1. Too Much 2. About Right
3. Too Little 4. No Opinion

	Under Grad	Post Grad
a) theoretical development	____	____
b) useful results	____	____
c) well-formulated problems	____	____
d) unstructured problems	____	____
e) mathematical modelling	____	____
f) applications to other fields	____	____
g) history & philosophy of math	____	____
h) social implications of math	____	____
i) lectures	____	____
j) seminars, group discussions	____	____
k) individual guidance	____	____
l) tutorials, problem sessions	____	____
m) independent study	____	____
n) original work	____	____
o) developing teaching ability	____	____
p) written assignments	____	____
q) examinations	____	____
r) teamwork	____	____

THANK YOU FOR YOUR COOPERATION IN COMPLETING THIS QUESTIONNAIRE.

D. Please express an opinion on the *DEGREE OF CONSIDERATION* that was given to each of the following in your own *MATHEMATICAL EDUCATION,* by inserting a *code number* corresponding to the following

CODED ANSWERS

1. Too Much 2. About Right 3. Too Little 4. No Opinion

	High School	Under-Graduate	Graduate
a) *ORIGINS* of mathematical concepts and theories	____	____	____
b) *INTERCONNECTIONS* between various branches of math	____	____	____
c) *LIMITATIONS* and *ABUSES* of mathematical methods	____	____	____
d) *COMMUNICATING* mathematical ideas to others	____	____	____
e) developing *COMPUTATIONAL SKILLS*	____	____	____
f) learning how to use the *MATHEMATICAL LITERATURE*	____	____	____
g) *CONTACT* with users of mathematics (invited talks, etc)	____	____	____
h) *CAREER OPPORTUNITIES* in the Mathematical Sciences	____	____	____
i) the *IMPACT* of mathematics on modern society	____	____	____
j) *CULTURAL ASPECTS* of mathematics	____	____	____
k) *CONTEMPORARY DEVELOPMENTS* in mathematics	____	____	____

APPENDIX I

Table 2 - Percent Response to the Mathematics Questionnaire by Degree Level and by Year of Graduation: All Classes - Mathematical Sciences - Canadian Universities[1]

Year of Graduation	1960[1]	1965[3]	1970	1971	1972	1973	All Years
Bachelors Degree							
Degrees awarded	138	263	699	911	1181	1138	4330
Number of respondents	25	84	229	253	333	352	1276
Percent response	18.1%	31.9%	32.8%	27.8%	28.2%	30.9%	29.5%
Masters Degree							
Degrees awarded	25	95	304	340	318	250	1332
Number of respondents	6	40	67	96	85	93	387
Percent response	24.0%	42.1%	22.0%	28.2%	26.7%	37.2%	29.1%
Doctoral Degree							
Degree awarded	45	182	47	77	90	75	516
Number of respondents	10	59	20	27	24	28	168
Percent response	22.2%	32.4%	42.6%	35.1%	26.7%	37.3%	32.6%
All Degrees							
Degrees awarded	208	540	1050	1323	1589	1463	6178
Number of respondents	41	183	316	376	442	473	1831
Percent response	19.7%	33.9%	30.1%	28.3%	27.8%	32.3%	29.6%

1 See note at end of Appendix I.
2 For PhD's 1960 refers to 1960 to 1964 inclusive.
3 For PhD's 1965 refers to 1965 to 1969 inclusive.

Note: The unadjusted response rate calculated above is simply the ratio of useable returns to the questionnaires mailed out (degrees awarded) x 100%.

Source: Primary research by the authors.

APPENDIX I

Table I.G - Current Job [2] Related to Mathematics by Degree[1] Level: All Classes - Mathematical Sciences - Canadian Universities

Level of last highest degree	B.A.	M.A.	Ph.D.
	%	%	%
(i) Job Related to Mathematics	80.2	89.7	97.5
(ii) Reasons for job not related to Mathematics:			
(a) preferred to do something other than math	7.1	3.7	1.9
(b) promoted out of mathematics position	0.9	1.5	0.6
(c) pay is better	1.4	0.0	0.0
(d) locational preference	0.6	0.4	0.0
(e) mathematical position not available	5.2	2.9	0.0
(f) other	3.7	1.8	0.0
(iii) No response	0.9	0.0	0.0
Total	100.0	100.0	100.0
Number of respondents	938	272	157

[2] Tabulation restricted to those working full-time.

Note: See general notes at the beginning of the Tables.

Source: Primary research by the authors.

APPENDIX I

Table II.B.1 - The First, Second and Third Principle Activity [1] of the Graduates' Present Job by Degree Level: All Classes - Mathematical Sciences - Canadian Universities [1]

	First Principle Activity			Second Principle Activity			Third Principle Act.		
Level of last highest degree	B.A.	M.A.	PhD	B.A.	M.A.	PhD	B.A.	M.A.	PhD
	%	%	%	%	%	%	%	%	%
Function									
a) applying routine mathematical procedures	15.7	8.8	3.8	12.3	10.7	0.0	7.2	4.0	0.0
b) supervising or evaluating mathematical work	3.2	3.7	0.6	9.8	6.3	2.5	5.0	3.7	1.9
c) advising non-mathematicians on math-related problems	4.4	3.7	2.5	6.1	8.5	1.3	6.6	9.2	6.4
d) investigating unstructured problems	13.9	19.9	2.5	9.5	5.1	2.5	4.8	3.3	0.0
e) doing basic or applied mathematical research	3.2	8.8	23.6	4.5	6.6	29.9	2.7	5.1	16.6
f) writing mathematical papers, books or reports	1.1	2.6	3.2	2.6	9.2	6.4	2.6	5.5	7.6
g) increasing specialized knowledge: reading, seminars, etc.	3.5	2.9	2.5	9.8	12.9	10.8	8.0	11.8	14.6
h) teaching mathematics at the post-graduate level	0.2	1.1	8.3	0.9	1.8	14.0	0.3	2.6	8.3
i) teaching mathematics at college or under-graduate level	6.8	18.4	40.8	1.4	5.1	16.6	1.1	1.5	8.9
j) other mathematical teaching	10.1	3.7	1.3	1.8	2.2	1.3	1.1	0.7	0.6
k) non-mathematical teaching	2.5	2.6	1.3	3.5	2.2	0.0	2.7	1.5	0.6
l) administration	5.4	2.9	1.9	7.1	7.7	5.1	5.7	8.1	7.0
m) other functions	17.2	10.7	1.9	6.9	6.3	1.3	5.4	4.0	0.6
No Response	12.9	10.3	5.7	23.9	15.4	8.3	47.2	39.0	26.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Number	938	272	157	938	272	157	938	272	157

[1] Some respondents reported only one principal activity.

Source: Primary research by the authors. (Table 1, Question II.B)

APPENDIX I

Table II.B.2 - Percent of Non-Negative Responses To and Corresponding Median Value of the Percent Distribution of Working Time for the Various Function of Question II.B: All Classes[1] - Mathematical Sciences - Canadian Universities

Function	Percent non-negative response			Median Percent time spent		
Level of last highest degree:	B.A.	M.A.	Ph.D.	B.A.	M.A.	Ph.D.
	%	%	%	%	%	%
a. applying routine mathematical procedures	55.7	47.1	13.4	22.0	18.0	11.3
supervising or evaluating mathematical work	33.3	34.6	37.6	10.2	10.4	9.0
c. advising non-mathematicians on math-related problems	35.2	41.5	29.3	8.6	8.5	7.7
d. investigating unstructured problems	38.9	40.1	12.7	19.0	19.5	10.3
e. doing basic or applied mathematical research	17.1	31.3	79.0	11.4	15.3	19.9
f. writing mathematical papers, books, or reports	16.1	34.2	66.2	10.3	10.9	9.9
g. increasing specialized knowledge: reading, seminars	45.1	58.5	67.5	9.6	10.0	11.8
h. teaching mathematics at post-graduate level	2.1	9.9	45.2	15.0	16.7	17.8
i. teaching mathematics at college or undergraduate level	8.0	29.0	73.2	44.0	41.4	34.1
j. other mathematics teaching	18.0	15.4	7.6	57.8	37.5	32.0
k. non-mathematics teaching	14.1	9.9	8.9	15.8	9.0	10.1
l. administration	40.3	48.9	56.1	10.3	10.4	9.7
m. other functions	51.3	43.0	17.8	39.2	24.1	8.3
Number of respondents	938	272	157	-	-	-

Source: Primary research by the authors. (Table 1, Question II.B)

APPENDIX I

Table II.A - Academic Qualifications Required in Current Job by Degree Level: All Classes - Mathematical Sciences - Canadian Universities[1]

Academic qualification required	(I) According to your employer			(II) In your own opinion		
Level of last highest degree:	B.A.	M.A.	Ph.D.	B.A.	M.A.	Ph.D.
	%	%	%	%	%	%
1. No degree required (technical training would suffice)	20.9	9.6	1.3	24.6	9.2	2.5
2. Bachelor's degree required						
(a) in the Mathematical Sciences	31.9	21.7	1.9	33.8	25.7	1.3
(b) not necessarily in the Mathematical Sciences	33.5	18.0	1.3	27.2	15.1	1.3
3. Graduate degree required						
(a) in the Mathematical Sciences	7.1	40.0	82.2	8.0	39.7	84.1
(b) not necessarily in the Mathematical Sciences	3.4	10.3	12.1	2.7	8.8	10.2
4. No response	3.2	0.4	1.3	3.7	1.5	0.6
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number	938	272	157	938	272	157

Source: Primary research by the authors. (Table 1, Question II.A)

APPENDIX I

Table II.F.1 - Satisfaction with the Work Environment by Degree Level: All Classess - Mathematical Sciences - Canadian Universities[1]

Level of Last Highest Degree	B.A. (N-938)		M.A. (N-272)		PhD (N-157)	
Aspect of job	Mean* Value	Modal*	Mean* Value	Modal*	Mean* Value	Modal*
a. congenial colleagues	64.9	70	63.4	70	59.6	70
b. good working conditions	60.6	60	60.4	70	61.4	70
c. good salary	56.6	60	57.3	60	58.2	50
d. job security	62.1	70	60.1	70	56.4	70
e. promotion prospects	52.6	60	50.5	60	50.2	60
f. novelty and variety of employment	55.6	60	56.1	60	52.6	50
g. utilization of training	51.3	60	52.1	50	61.5	70
h. personal satisfaction	59.9	60	60.8	60	63.7	70
i. scope for individual initiative	60.2	60	62.0	70	63.9	60
j. intellectual stimulation	53.1	60	55.3	60	58.6	60
k. prestige inside peer group	52.5	50	54.2	60	55.0	60
l. prestige outside peer group	53.5	60	54.8	60	53.4	60
m. social welfare	53.2	50	53.1	60	51.2	50

* The mean value" and "modal value" represents a scale between 20 and 80. From 20-40 represents "hardly at all", 41-60 represents "moderately", and 61-80, "to a high degree".

Source: Primary research by the authors. (Table 1, Question II.F)

APPENDIX I

Table II.F.2 - Detailed Distribution, Satisfaction Scores: All Classes, B.Sc. (Hon.) - Mathematical Sciences - Canadian Universities (N-938)[1]

Arbitrary Satisfaction Scale*	20-30	31-40	41-50	51-60	61-70	71-80	No response
Aspects of Job							
a. congenial colleagues	2.9	2.3	11.1	24.0	35.8	22.3	1.5
b. good working conditions	5.0	7.5	15.0	27.3	27.4	16.7	1.0
c. good salary	8.2	9.9	23.4	25.0	21.1	11.3	1.1
d. job security	8.6	6.1	11.3	20.5	24.6	27.7	1.2
e. promotion prospects	19.2	10.9	18.1	21.2	17.6	11.7	1.2
f. novelty and variety of employment	12.5	11.9	18.7	22.0	19.1	14.4	1.4
g. utilization of training	20.8	13.0	19.4	20.4	11.5	12.8	1.2
h. personal satisfaction	6.3	7.7	14.4	24.3	22.6	16.8	8.0
i. scope for individual initiative	6.5	7.5	16.8	23.7	24.1	20.0	1.1
j. intellectual stimulation	14.4	12.2	22.7	24.0	16.5	9.2	1.2
k. prestige inside peer group	12.2	12.0	28.1	22.0	15.2	6.0	4.5
l. prestige outside peer group	12.9	9.5	24.4	25.1	15.6	7.6	4.8
m. social usefulness	15.4	13.3	19.3	19.1	17.7	11.5	3.7

*Scores 20-40 represent "hardly at all", 41-60 "moderately" and 61-80 "to a high degree".

Source: Primary research by the authors. (Table 1, Question II.F)

Total - 100.

APPENDIX I

Table II.F.3 - Detailed Distribution of Satisfaction Scores, All Classes, M.A. - Mathematical Sciences - Canadian Universities (N-272)[1]

Arbitrary Satisfaction Scale*	20-30	31-40	41-50	51-60	61-70	71-80	No response
Percentage Distribution							
Aspects of Job							
a. congenial colleagues	3.3	5.3	14.0	21.0	34.5	20.2	1.8
b. good working conditions	5.5	8.1	15.1	23.9	29.9	16.0	1.5
c. good salary	8.1	6.6	24.0	26.4	22.8	11.3	1.1
d. job security	11.4	5.5	13.6	19.9	25.4	23.2	1.1
e. promotion prospects	22.4	12.8	16.9	18.0	18.8	8.5	2.6
f. novelty and variety of employment	11.3	11.0	17.3	25.0	18.0	14.7	2.6
g. utilization of training	18.4	13.2	21.7	18.7	14.7	12.2	1.1
h. personal satisfaction	4.4	7.0	14.3	26.4	23.5	16.1	8.1
i. scope for individual initiative	6.7	5.9	11.8	21.4	29.4	23.2	1.8
j. intellectual stimulation	11.8	10.3	20.3	26.5	16.1	12.5	2.6
k. prestige inside peer group	9.2	16.2	19.8	24.3	15.8	9.9	4.8
l. prestige outside peer group	9.6	10.3	22.4	27.2	17.3	7.7	5.5
m. social usefulness	15.2	12.1	20.3	22.5	15.1	11.0	4.0

*Scores 20-40 represent "hardly at all", 41-60 "moderately" and 61-80 "to a high degree".

Source: Primary research by the authors. (Table 1, Question II.F)

Total - 100

APPENDIX I

Table II.F.4 - Detailed Distribution of Satisfaction Scores: All Classes, PhD - Mathematical Sciences - Canadian Universities (N-157) [1]

Arbitrary Satisfaction Scale*	20-30	31-40	41-50	51-60	61-70	71-80	No response
Percentage Distribution							
Aspects of Job							
a. congenial colleagues	6.4	4.5	25.5	19.1	26.8	14.6	3.2
b. good working conditions	3.8	7.6	16.5	20.3	36.9	12.7	1.9
c. good salary	4.5	7.0	30.0	21.6	22.9	12.1	1.9
d. job security	16.6	9.5	12.1	17.2	21.6	20.3	2.5
e. promotion prospects	22.7	10.2	19.8	24.2	12.8	7.6	3.8
f. novelty and variety of employment	16.5	8.3	27.4	20.4	14.0	10.2	3.2
g. utilization of training	7.0	4.5	20.4	15.2	24.8	24.8	2.5
h. personal satisfaction	1.9	6.3	12.1	19.1	29.3	21.0	10.2
i. scope for individual initiative	3.8	3.8	13.4	24.2	14.2	27.3	3.2
j. intellectual stimulation	8.3	10.8	15.9	24.9	17.1	20.4	2.5
k. prestige inside peer group	10.9	16.8	15.2	29.3	18.5	6.4	8.9
l. prestige outside peer group	9.5	15.3	18.4	30.0	12.1	7.0	7.6
m. social usefulness	13.4	14.0	24.9	19.8	14.7	4.5	8.9

*Scores 20-40 represent "hardly at all", 41-60 "moderately" and 61-80 "to a high degree".

Source: Primary research by the authors. (Table 1, Question II.F)

Total - 100

APPENDIX I

Table II.G.1 - Opinions of High School Mathematics Subject Areas by Province[1] of High School: All Classes, B.Sc. (Hon.) - Mathematical Sciences and Canadian Universities

High School Algebra

Provinces:	Maritimes	Quebec	Ontario	Manitoba	Saskatchewan	Alberta	B.C.
Degree of Emphasis Placed on this subject area:	%	%	%	%	%	%	%
About Right	75.2	80.9	82.6	82.1	85.9	80.3	81.5
Too Much	5.1	5.1	2.2	1.5	0.0	5.3	6.2
Too Little	17.9	13.4	13.2	14.9	12.7	13.2	10.8
No Opinion	1.7	0.6	2.0	1.5	1.4	1.3	1.5
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	117	157	598	67	71	76	65

High School Geometry

Provinces	Maritimes	Quebec	Ontario	Manitoba	Saskatchewan	Alberta	B.C.
Degree of Emphasis Placed on this subject area:	%	%	%	%	%	%	%
About Right	71.8	74.5	73.3	70.3	81.9	68.4	63.0
Too Much	13.7	11.5	11.6	14.9	6.9	11.4	18.5
Too Little	12.8	12.7	12.7	12.2	9.7	20.3	14.8
No Opinion	1.7	1.3	2.5	2.7	1.4	0.0	3.7
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	117	157	606	74	72	79	54

[1] The Maritime Provinces were grouped due to insufficient Provincial response.

Source: Primary Research by the authors. (Table 1, Question II.G)

Trigonometry

Provinces:	Maritimes	Quebec	Ontario	Manitoba	Saskatchewan	Alberta	B.C.
Degree of Emphasis placed on this subject area:	%	%	%	%	%	%	%
About Right	65.8	73.1	76.4	71.6	77.5	72.4	67.7
Too Much	6.0	8.3	7.4	4.5	2.8	1.3	9.2
Too Little	25.6	16.0	13.5	20.9	15.5	23.7	18.5
No Opinion	2.6	2.6	2.7	3.0	4.2	2.6	4.6
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	117	156	598	67	71	76	65

Elementary Set Theory

Provinces:	Maritimes	Quebec	Ontario	Manitoba	Saskatchewan	Alberta	B.C.
	%	%	%	%	%	%	%
Degree of Emphasis placed on this subject area:							
	28.2	38.6	48.8	39.7	47.1	47.3	43.3
Too Much	3.4	5.2	3.2	5.9	2.9	2.7	4.5
Too Little	63.2	50.3	35.9	44.1	48.6	47.3	47.8
No Opinion	5.1	5.9	7.0	10.3	1.4	2.7	4.5
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	117	153	596	68	70	74	67

APPENDIX I

Table II.G.2 - (i) Opinions of University Mathematics Subject Areas, and (ii) Opinions of the Importance of this Education in Jobs: All Classes, B.Sc. (Hon.) - Mathematical Sciences - Canadian Universities[1]

	1. Finite Mathemtics		
	Point Set Topology	Algebraic Topology	Manifold Topology
(i)	%	%	%
About Right	46.6	39.2	42.8
Too Much	6.0	2.3	3.5
Too Little	36.7	31.5	37.1
No Opinion	10.7	26.9	16.7
Total	100.0	100.0	100.0
Number of Respondents	1,147	1,117	1,128
(ii)	%	%	%
Great Importance	14.5	6.9	13.4
Moderate Importance	29.8	25.1	30.5
Little Importance	24.9	23.5	23.0
No Importance	30.7	44.5	33.0
Total	100.0	100.0	100.0
Number of Respondents	999	956	982

	2. University Algebra		
(i)	Linear & Dynamic Programming	Variational Principles	Control Theory
	%	%	%
About Right	53.7	68.9	47.3
Too Much	10.7	11.9	7.1
Too Little	22.4	16.1	27.8
No Opinion	13.2	3.1	17.8
Total	100.0	100.0	100.0
Number of Respondents	1,155	1,177	1,128
(ii)	%	%	%
Great Importance	5.1	14.7	10.0
Moderate Importance	18.6	27.0	22.0
Little Importance	28.7	24.8	23.2
No Importance	47.5	33.5	44.8
Total	100.0	100.0	100.0
Number of Respondents	999	1,018	980

(i) Degree of emphasis placed on this subject area
(ii) Importance of this subject area in your job
Source: Primary research by the authors. (Table 1, Question II.G)

	3. University Geometry		
	Point Set Topology	Algebraic Topology	Manifold Topology
(i)	%	%	%
About Right	37.2	32.0	30.7
Too Much	5.6	5.8	5.6
Too Little	24.7	25.4	24.6
No Opinion	32.5	36.8	39.2
Number of Respondents	1,065	1,053	1,057
(ii)			
Great Importance	4.4	2.9	2.0
Moderate Importance	11.2	6.2	6.6
Little Importance	21.1	20.4	20.4
No Importance	63.3	70.5	70.9
Number of Respondents	9166	906	911

	4. Analysis		
	Linear & Dynamic Programming	Variational Principles	Control Theory
(i)	%	%	%
About Right	83.9	72.4	60.0
Too Much	5.5	11.7	6.7
Too Little	9.1	11.0	27.2
No Opinion	1.5	4.9	6.1
Number of Respondents	1,193	1,171	1.156
(ii)			
Great Importance	19.5	10.4	7.0
Moderate Importance	26.6	19.2	18.0
Little Importance	22.6	24.5	24.9
No Importance	31.3	45.9	50.1
Number of Respondents	1,036	1,010	999

	5. Topology		
	Point Set Topology	Algebraic Topology	Manifold Topology
(i)	%	%	%
About Right	31.9	24.5	16.0
Too Much	6.4	6.3	4.1
Too Little	17.3	20.0	19.6
No Opinion	44.5	49.2	60.3
Total	100.0	100.0	100.0
Number of Respondents	1,019	997	979
(ii)			
Great Importance	4.0	2.5	2.6
Moderate Importance	7.7	5.2	2.9
Little Importance	12.2	12.1	11.4
No Importance	76.1	80.2	83.1
Total	100.0	100.0	100.0
Number of Respondents	870	850	834

	6. Optimization		
(i)	Linear & Dynamic Programming	Variational Principles	Control Theory
	%	%	%
About Right	31.3	17.2	15.0
Too Much	1.3	1.0	0.9
Too Little	39.4	32.0	35.4
No Opinion	27.9	49.7	48.7
Total	100.0	100.0	100.0
Number of Respondents	1,045	977	983
(ii)			
Great Importance	11.2	6.0	5.7
Moderate Importance	24.7	14.2	16.7
Little Importance	20.5	15.3	15.6
No Importance	43.6	64.4	62.0
Total	100.0	100.0	100.0
Number of Respondents	908	829	839

	7. Computer Science		
	Computer Programming	Simulation Techniques	Systems Analysis
(i)	%	%	%
About Right	54.7	24.9	18.2
Too Much	5.8	1.9	1.9
Too Little	33.7	53.0	60.8
No Opinion	5.7	20.2	19.1
Total	100.0	100.0	100.0
Number of Respondents	1,166	1,099	1,100
(ii)			
Great Importance	50.0	24.6	39.0
Moderate Importance	22.7	26.9	21.5
Little Importance	10.5	17.0	11.2
No Importance	16.8	31.5	28.2
Total	100.0	100.0	100.0
Number of Respondents	1,025	955	961

	8. Probability and Statistics				
(i)	Elementary Statistics	Other Stat.	Stochastic Processes	Queuing Theory	Probability Theory
	%	%	%	%	%
About Right	64.2	46.1	30.3	30.2	59.2
Too Much	2.5	1.9	2.5	2.2	4.0
Too Little	28.9	28.8	30.2	33.2	25.6
No Opinion	4.4	23.1	37.0	34.5	11.1
Total	100.0	100.0	100.0	100.0	100.0
Number of Respondents	1,171	1,034	1,012	1,021	1,104
(ii)					
Great Importance	25.4	17.1	8.3	8.0	16.2
Moderate Importance	36.3	26.1	19.6	20.1	31.8
Little Importance	20.3	19.4	18.6	19.9	20.5
No Importance	18.0	37.4	53.5	52.0	31.4
Total	100.0	100.0	100.0	100.0	100.0
Number of Respondents	1,030	896	866	879	949

	9. Applications of Mathematics		
	Actuarial Science	Operations Research	Mathematical Physics
(i)	%	%	%
About Right	24.8	28.5	35.3
Too Much	2.5	1.3	5.8
Too Little	26.4	39.9	19.3
No Opinion	46.3	30.2	39.6
Total	100.0	100.0	100.0
Number of Respondents	953	979	958
(ii)			
Great Importance	14.0	12.9	5.6
Moderate Importance	11.0	24.4	11.6
Little Importance	12.6	17.9	14.4
No Importance	62.3	44.8	68.5
Total	100.0	100.0	100.0
Number of Respondents	815	853	822

APPENDIX I

Table II.G.3 - (i) Opinions of University Mathematics Subject Areas and (ii) Opinions of the Importance of this Education in Jobs: Classes of 1960/65, 1970, 1971, 1972 and 1973, B.Sc.(Hon.) - Mathematical Sciences - Canadian Universities[1]

	1. Finite Mathematics - a) Numerical Analysis					
Year of Graduation:	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	44.7	43.4	46.9	43.6	51.7	46.6
Too Much	4.9	5.7	5.3	6.5	6.7	6.0
Too Little	41.7	36.3	36.7	38.8	33.3	36.7
No Opinion	8.7	14.6	11.1	11.0	8.3	10.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	103	212	226	291	315	1,147
(ii)						
Great Importance	18.9	15.4	13.7	12.2	15.1	14.5
Moderate Importance	31.6	28.7	26.8	31.7	30.6	29.8
Little Importance	23.0	20.5	31.2	23.6	24.8	24.9
No Importance	24.0	35.4	28.3	32.5	29.5	30.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	195	205	246	258	999

(i) Degree of emphasis placed on this subject area
(ii) Importance of this subject area in your job

Source: Primary research by the authors. (Table 1, Question II.G)

	1. Finite Mathematics - b) Combinaterics					
Year of Graduation:	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	31.7	41.5	38.9	41.4	38.4	39.2
Too Much	5.9	2.9	1.9	1.8	1.6	2.3
Too Little	36.6	29.8	29.6	33.0	31.0	31.5
No Opinion	25.7	25.9	29.6	23.9	29.0	26.9
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	101	205	216	285	310	1,117
(ii)						
Great Importance	9.8	5.4	7.3	8.8	4.8	6.9
Moderate Importance	30.4	23.8	25.5	22.7	26.1	25.1
Little Importance	21.7	22.2	22.9	23.9	25.3	23.5
No Importance	38.1	48.6	44.3	44.5	43.8	44.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	185	192	238	249	956

Year of Graduation:	1. Finite Mathematics - c) Graphs and Networks					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	34.0	43.1	41.2	43.9	45.5	42.8
Too Much	6.0	2.9	5.0	2.4	2.9	3.5
Too Little	42.0	36.3	35.3	39.8	34.7	37.1
No Opinion	18.0	17.6	18.6	13.8	16.9	16.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	100	204	221	289	314	1,128
(ii)						
Great Importance	16.0	15.5	13.9	13.7	10.3	13.4
Moderate Importance	29.8	25.1	31.8	31.0	33.3	30.5
Little Importance	28.7	22.5	20.4	22.6	23.8	23.0
No Importance	25.5	36.9	33.8	32.7	32.5	33.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	187	201	248	252	982

Year of Graduation:	2. University Algebra - a) Number Theory					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	57.0	55.5	58.1	52.5	49.7	53.7
Too Much	7.0	10.5	11.5	9.4	12.8	10.7
Too Little	29.0	16.3	17.2	27.4	23.4	22.4
No Opinion	7.0	17.7	13.2	11.0	14.1	13.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	100	209	227	299	320	1,155
(ii)						
Great Importance	6.3	6.3	4.4	5.9	3.5	5.1
Moderate Importance	27.4	21.1	17.2	19.3	14.0	18.6
Little Importance	30.5	26.3	30.0	25.2	32.3	28.7
No Importance	35.8	46.3	48.3	49.6	50.2	47.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	190	203	254	257	999

Year of Graduation:	2. Finite Mathematics - b) Linear Algebra					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	77.5	65.9	70.8	70.3	65.5	68.9
Too Much	4.9	12.3	9.4	12.2	15.2	11.9
Too Little	16.7	16.6	14.6	15.8	16.8	16.1
No Opinion	1.0	5.2	5.2	1.7	2.4	3.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	102	211	233	303	328	1,177
(ii)						
Great Importance	28.7	15.1	14.8	9.3	14.7	14.7
Moderate Importance	38.3	26.0	26.8	26.7	24.2	27.0
Little Importance	16.0	24.0	26.8	26.7	24.9	24.8
No Importance	17.0	34.9	31.6	37.2	36.2	33.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	192	209	258	265	1,018

Year of Graduation:	2. Finite Mathematics - c) Boolean Algebra					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	46.4	50.5	44.1	49.0	46.0	47.3
Too Much	4.1	7.9	9.0	6.5	6.7	7.1
Too Little	30.9	22.3	26.6	28.4	30.8	27.8
No Opinion	18.6	19.3	20.3	16.1	16.5	17.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	97	202	222	292	315	1,128
(ii)						
Great Importance	10.9	6.6	9.1	11.2	11.6	10.0
Moderate Importance	26.1	25.7	16.7	18.5	25.6	22.0
Little Importance	25.1	21.9	28.3	20.1	21.7	23.2
No Importance	35.9	45.9	46.0	50.2	41.1	44.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	183	198	249	258	980

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
3. University Geometry - a) Solid Geometry						
(i)						
About Right	40.4	46.4	35.0	38.6	30.2	37.2
Too Much	15.2	2.1	7.9	4.9	3.8	5.6
Too Little	25.3	19.1	24.3	25.8	27.5	24.7
No Opinion	19.2	32.5	32.7	30.7	38.5	32.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	194	214	267	291	1,065
(ii)						
Great Importance	9.8	5.6	2.6	3.5	3.4	4.4
Moderate Importance	15.2	10.2	14.3	10.6	8.6	11.2
Little Importance	30.4	19.8	22.2	19.5	19.0	21.1
No Importance	44.6	64.4	60.8	66.4	69.0	63.3
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	177	189	226	232	916

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
3. University Geometry - b) Differential Geometry						
(i)						
About Right	35.1	38.5	25.4	35.1	28.6	32.0
Too Much	9.3	2.1	8.6	6.0	4.8	5.8
Too Little	27.8	21.9	27.3	24.9	25.9	25.4
No Opinion	27.8	37.5	38.8	34.0	40.7	36.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	97	192	209	265	290	1,053
(ii)						
Great Importance	2.2	2.9	2.2	3.1	3.5	2.9
Moderate Importance	6.5	7.5	5.4	5.8	6.1	6.2
Little Importance	33.7	19.0	24.9	17.4	15.6	20.4
No Importance	57.6	70.7	67.6	73.7	74.9	70.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	174	185	224	231	906

Year of Graduation:	3. University Geometry - c) Projective Geometry					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	38.8	32.3	25.0	32.8	28.9	30.7
Too Much	10.2	3.6	8.0	5.6	3.5	5.6
Too Little	24.5	26.0	23.6	24.6	24.4	24.6
No Opinion	26.5	38.0	43.4	36.9	43.2	39.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	98	192	212	268	287	1,057
(ii)						
Great Importance	3.2	2.3	1.6	1.8	1.7	2.0
Moderate Importance	10.8	8.6	5.3	5.8	5.2	6.6
Little Importance	30.1	18.4	22.8	17.3	19.2	20.4
No Importance	55.9	70.7	70.4	75.2	73.8	70.9
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	93	174	189	226	229	911

	4. Analysis - a) Elementary Calculus					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	91.4	86.6	83.4	80.9	82.9	83.9
Too Much	3.8	3.7	5.5	6.5	6.4	5.5
Too Little	3.8	7.9	10.6	10.7	8.8	9.1
No Opinion	1.0	1.9	0.4	1.9	1.8	1.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	105	216	235	309	328	1,193
(ii)						
Great Importance	34.4	19.7	15.6	18.0	18.6	19.5
Moderate Importance	28.1	26.8	30.2	23.7	26.1	26.6
Little Importance	26.0	24.2	23.6	20.3	21.6	22.6
No Importance	11.5	29.3	30.7	38.0	33.7	31.3
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	96	198	212	266	264	1,036

4. Analysis - b) Advanced Calculus

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	82.7	76.0	69.4	74.6	67.6	72.4
Too Much	6.7	11.1	12.9	8.9	15.4	11.7
Too Little	9.6	9.1	12.9	9.6	12.7	11.0
No Opinion	2.9	3.8	4.7	6.9	4.3	4.9
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	104	208	232	303	324	1,171
(ii)						
Great Importance	18.1	11.5	7.7	7.4	12.0	10.4
Moderate Importance	29.8	16.8	22.5	15.6	18.1	19.2
Little Importance	28.7	27.7	25.4	20.6	23.6	24.5
No Importance	23.4	44.0	44.5	56.4	46.3	45.9
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	191	209	257	259	1,010

4. Analysis - c) Differential Equations

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	71.4	62.7	62.8	59.7	52.8	60.0
Too Much	3.8	6.7	8.8	6.0	6.6	6.7
Too Little	22.9	27.3	22.6	27.2	32.1	27.2
No Opinion	1.9	3.3	5.8	7.0	8.5	6.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	105	209	226	298	318	1,156
(ii)						
Great Importance	9.6	8.9	5.4	5.8	7.1	7.0
Moderate Importance	27.7	16.8	20.2	14.0	17.7	18.0
Little Importance	31.9	26.2	24.6	23.3	23.2	24.9
No Importance	30.9	48.2	49.8	56.8	52.0	50.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	191	203	257	254	999

	5. Topology - a) Point Set Topology					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	35.5	38.7	33.3	30.6	26.2	31.9
Too Much	9.7	7.3	5.1	6.6	6.1	6.4
Too Little	29.0	14.7	14.1	17.8	16.8	17.3
No Opinion	28.0	39.3	47.5	45.0	50.9	44.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	93	191	198	258	279	1,019
(ii)						
Great Importance	6.9	3.6	3.9	4.6	2.7	4.0
Moderate Importance	17.2	9.5	5.0	5.6	6.8	7.7
Little Importance	18.4	12.5	14.0	11.1	9.1	12.2
No Importance	57.5	74.4	77.1	78.7	81.4	76.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	87	168	179	216	220	870

	5. Topology - b) Algebraic Topology					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	25.0	29.3	24.2	25.3	20.4	24.5
Too Much	8.7	7.4	4.1	5.6	6.9	6.3
Too Little	31.5	19.1	16.5	18.5	20.4	20.0
No Opinion	34.8	44.1	55.2	50.6	52.2	49.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	188	194	249	274	997
(ii)						
Great Importance	1.2	1.2	2.9	3.8	2.3	2.5
Moderate Importance	9.4	6.6	2.9	4.8	4.7	5.2
Little Importance	23.5	12.7	13.1	11.0	7.5	12.1
No Importance	65.9	79.5	31.1	80.5	85.5	80.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	85	166	175	210	214	850

6. Topology - c) Manifold Theory

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	12.5	20.2	13.7	16.3	15.8	16.0
Too Much	4.5	3.8	4.2	4.1	4.0	4.1
Too Little	35.2	18.0	15.8	19.6	18.3	19.6
No Opinion	47.7	57.9	66.3	60.0	61.9	60.3
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	183	190	245	273	979
(ii)						
Great Importance	1.2	1.8	1.8	3.9	3.3	2.6
Moderate Importance	2.5	6.1	1.8	1.0	3.3	2.9
Little Importance	22.2	11.0	12.9	10.2	7.5	11.4
No Importance	74.1	81.0	83.5	85.0	86.0	83.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	81	163	170	206	214	834

7. Optimization - a) Linear & Dynamic Programming

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	12.8	30.1	31.2	33.7	35.9	31.3
Too Much	1.1	1.1	1.5	1.9	1.0	1.3
Too Little	57.4	35.5	38.5	39.6	36.6	39.4
No Opinion	28.7	33.3	28.8	24.8	26.6	27.9
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	186	205	270	290	1,045
(ii)						
Great Importance	15.7	8.7	9.7	11.5	12.3	11.2
Moderate Importance	27.0	23.8	25.9	23.5	24.6	24.7
Little Importance	21.3	15.1	23.8	20.5	21.5	20.5
No Importance	36.0	52.3	40.5	44.4	41.7	43.6
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	89	172	185	234	228	908

	7. Optimization - b) Variational Principles					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	11.4	20.0	14.7	19.0	17.4	17.2
Too Much	2.3	1.1	1.0	0.3	0.7	1.0
Too Little	45.5	32.6	29.8	29.1	31.5	32.0
No Opinion	40.9	46.3	54.5	51.0	50.4	49.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	175	191	247	276	977
(ii)						
Great Importance	10.7	4.4	5.9	4.8	6.7	6.0
Moderate Importance	17.9	14.6	12.4	13.4	14.8	14.2
Little Importance	17.9	11.4	16.6	16.3	15.3	15.3
No Importance	53.6	69.6	65.1	65.6	63.2	64.4
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	84	158	169	209	209	829

	7. Optimization - c) Control Theory					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	11.4	16.5	10.5	17.9	15.5	15.0
Too Much	1.1	1.1	1.0	0.4	1.1	0.9
Too Little	48.9	36.4	33.5	32.7	34.3	35.4
No Opinion	38.6	46.0	55.0	49.0	49.1	48.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	176	191	251	277	983
(ii)						
Great Importance	8.2	3.8	4.8	5.6	7.1	5.7
Moderate Importance	18.8	13.1	13.1	17.2	20.9	16.7
Little Importance	21.2	15.0	17.3	15.3	12.8	15.6
No Importance	51.8	68.1	64.9	61.9	59.2	62.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	85	160	168	215	211	839

8. Computer Science - c) Computer Programming

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	23.8	50.5	50.2	61.3	64.2	54.7
Too Much	3.0	5.6	5.3	5.7	7.4	5.8
Too Little	65.3	35.5	38.3	28.3	24.4	33.7
No Opinion	7.9	8.4	6.2	4.7	4.0	5.7
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	101	214	227	300	324	1,166
(ii)						
Great Importance	37.9	47.3	48.1	49.2	58.6	50.0
Moderate Importance	32.6	24.9	25.2	17.9	20.3	22.7
Little Importance	15.8	11.9	8.7	13.7	5.7	10.5
No Importance	13.7	15.9	18.0	19.1	15.3	16.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	201	206	262	261	1,025

8. Computer Science - c) Simulation Technique

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	14.1	23.7	21.7	28.9	27.8	24.9
Too Much	0.0	2.1	1.8	1.7	2.6	1.9
Too Little	66.0	51.0	55.3	53.7	47.4	53.0
No Opinion	19.2	23.2	21.2	15.7	22.2	20.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	194	217	287	302	1,099
(ii)						
Great Importance	32.3	23.6	22.4	23.0	25.8	24.6
Moderate Importance	25.8	27.5	28.1	24.6	28.4	26.9
Little Importance	14.0	12.6	19.8	18.3	17.8	17.0
No Importance	28.0	36.3	29.7	34.1	28.0	31.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	93	182	192	252	236	955

8. Computer Science - c) Systems Analysis

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	10.0	14.0	15.8	18.7	24.7	18.2
Too Much	0.0	2.6	0.9	1.4	3.2	1.9
Too Little	71.0	59.6	62.8	65.1	52.9	60.8
No Opinion	19.0	23.8	20.5	14.8	19.2	19.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	100	193	215	284	308	1,100
(ii)						
Great Importance	39.4	40.1	38.7	36.5	40.8	39.0
Moderate Importance	22.3	21.4	18.8	20.5	24.5	21.5
Little Importance	14.9	6.0	12.6	11.2	12.7	11.2
No Importance	23.4	32.4	29.8	31.7	22.0	23.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	182	191	249	245	961

9. Probability and Statistics - a) Elementary Statistics

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	58.7	67.8	63.5	60.6	67.6	64.2
Too Much	1.0	1.9	2.1	2.0	4.0	2.5
Too Little	35.6	26.0	30.0	33.4	23.8	28.9
No Opinion	4.8	4.3	4.3	4.0	4.6	4.4
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	104	208	233	302	324	1,171
(ii)						
Great Importance	32.0	24.0	28.0	25.1	22.4	25.4
Moderate Importance	39.2	37.2	34.1	36.5	36.1	36.3
Little Importance	16.5	22.4	19.0	19.0	22.4	20.3
No Importance	12.4	16.3	19.0	19.4	19.0	18.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	97	196	211	263	263	1,030

	9. Probability and Statistics - b) Other Statistics					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	46.2	47.0	48.0	41.1	49.0	46.1
Too Much	1.1	1.1	2.5	1.5	2.8	1.9
Too Little	35.5	28.6	26.0	33.7	24.1	28.8
No Opinion	17.2	23.2	23.5	23.7	24.1	23.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	93	185	200	270	286	1,034
(ii)						
Great Importance	23.0	16.2	18.0	17.7	14.2	17.1
Moderate Importance	27.6	24.3	26.4	25.4	27.4	26.1
Little Importance	21.8	19.1	16.3	20.3	20.4	19.4
No Importance	27.6	40.5	39.3	36.6	38.1	37.4
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	87	173	178	232	226	896

	9. Probability and Statistics - c) Stochastic Processes					
	1960/65	1970	1971	1972	1973	ALL
	%	%	%	%	%	%
(i)						
About Right	32.3	27.5	29.8	34.2	23.2	30.3
Too Much	2.2	2.2	3.5	0.4	4.0	2.5
Too Little	43.0	29.8	26.3	30.1	29.2	30.2
No Opinion	22.6	40.4	40.4	35.3	38.6	37.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	93	178	198	266	277	1,012
(ii)						
Great Importance	10.3	11.6	6.3	6.2	8.8	8.3
Moderate Importance	29.9	14.0	19.0	18.6	21.4	19.6
Little Importance	19.5	12.8	19.5	19.0	21.4	18.6
No Importance	40.2	61.6	55.2	56.2	48.4	53.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	87	164	174	226	215	866

9. Probability and Statistics - d) Queuing Theory

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	25.0	30.6	28.9	34.1	28.7	30.2
Too Much	2.2	2.2	3.0	1.1	2.5	2.2
Too Little	47.8	27.2	31.0	32.6	34.4	33.2
No Opinion	25.0	40.0	37.1	32.2	34.4	34.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	92	180	197	270	282	1,021
(ii)						
Great Importance	9.3	6.5	9.1	7.0	8.6	8.0
Moderate Importance	25.6	18.5	18.3	17.5	23.4	20.1
Little Importance	25.6	13.1	20.0	19.7	23.0	19.9
No Importance	39.5	61.9	52.6	55.7	45.0	52.0
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	86	168	175	228	222	879

9. Probability and Statistics - e) Probability Theory

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	53.2	59.6	55.3	61.5	61.4	59.2
Too Much	3.2	3.6	4.2	3.8	4.5	4.0
Too Little	37.2	25.4	27.0	25.8	21.2	25.6
No Opinion	6.4	11.4	13.5	8.9	12.9	11.1
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	94	193	215	291	311	1,104
(ii)						
Great Importance	29.5	14.5	16.0	14.9	14.2	16.2
Moderate Importance	29.5	34.6	28.4	33.1	32.1	31.8
Little Importance	21.6	17.3	23.7	18.1	22.5	20.5
No Importance	19.3	33.5	32.0	33.9	31.3	31.4
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	179	194	248	240	949

10. Applications of Mathematics - a) Actuarial Science

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	33.0	20.0	22.9	23.1	27.7	24.8
Too Much	6.8	4.0	3.9	0.8	0.7	2.5
Too Little	26.1	26.3	30.7	26.0	24.0	26.4
No Opinion	31.8	49.7	42.5	50.0	47.6	46.3
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	86	175	179	242	271	953
(ii)						
Great Importance	17.3	10.7	16.6	12.0	15.2	14.0
Moderate Importance	8.6	11.3	8.6	9.6	15.2	11.0
Little Importance	17.3	10.7	14.1	13.9	9.8	12.6
No Importance	56.8	67.3	60.7	64.4	59.8	62.3
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	81	159	163	208	204	815

10. Applications of Mathematics - b) Operations Research

	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	15.9	28.2	28.6	29.3	31.8	28.5
Too Much	2.3	1.1	1.1	1.6	1.1	1.3
Too Little	58.0	41.2	38.9	38.6	35.4	39.9
No Opinion	23.9	29.4	31.4	30.5	31.8	30.2
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	177	185	249	280	979
(ii)						
Great Importance	19.0	11.1	13.9	10.4	13.6	12.9
Moderate Importance	28.6	24.1	22.9	22.6	25.9	24.4
Little Importance	19.0	14.2	18.1	18.1	20.0	17.9
No Importance	33.4	50.6	45.2	48.9	40.5	44.8
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	84	162	166	221	220	853

10. Applications of Mathematics - c) Mathematical Physics	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
(i)						
About Right	48.9	35.1	36.8	33.3	31.7	35.3
Too Much	14.8	5.2	5.4	4.6	4.8	5.8
Too Little	15.9	14.9	18.9	24.6	18.8	19.3
No Opinion	20.5	44.8	38.9	37.5	44.6	39.6
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	88	174	185	240	271	958
(ii)						
Great Importance	7.2	7.7	6.0	2.4	6.3	5.6
Moderate Importance	10.8	11.6	11.3	13.0	10.6	11.6
Little Importance	31.3	10.3	13.1	14.4	11.5	14.4
No Importance	50.6	70.3	69.6	25.8	71.6	68.5
TOTAL	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	83	155	168	208	208	822

APPENDIX I

Table III.C.1.1 - Opinions regarding Content and Methodology of University Mathematical Education: All Classess, B.Sc. (Hon.) - Mathematical Sciences - Canadian Universities [1]

	Opinions				
Content and Methodology	About Right %	Too Much %	Too Little %	No Opinion %	Number of Res.
1. Theoretical development	52.5	35.6	9.0	2.9	1,215
2. Useful results	31.7	1.2	60.8	6.4	1,200
3. Well-formulated problems	58.9	6.2	27.1	7.7	1,190
4. Unstructured problems	30.0	5.6	41.0	23.4	1,165
5. Mathematical modelling	24.1	1.5	52.2	22.3	1,167
6. Applications to other fields	24.3	0.8	66.2	8.6	1,197
7. History and philosophy of math	29.9	4.5	47.0	18.6	1,182
8. Social Implications of math	13.0	1.6	61.4	24.0	1,169
9. Lectures	61.9	29.1	4.0	5.0	1,201
10. Seminars, group discussions	29.5	2.0	58.2	10.3	1,185
11. Individual guidance	42.4	0.8	49.5	7.4	1,193
12. Tuorials, problem sessions	52.5	4.3	36.3	6.9	1,194
13. Independent study	59.6	6.6	25.6	8.2	1,181
14. Original work	30.6	0.6	50.2	18.6	1,171
15. Developing teaching ability	19.8	0.8	52.7	26.7	1,167
16. Written assignments	73.0	10.1	12.1	4.8	1,202
17. Examinations	64.3	29.5	3.2	3.1	1,206
18. Teamwork	24.6	2.4	56.3	16.7	1,171

Source: Primary research by the authors (Table 1, Question III.C)

Total - 100

APPENDIX I

Table III.C.1-2 - Opinions Regarding Content and Methodology of University Mathematical Education: B.Sc. (Hon.) - Mathematical Sciences - Canadian Universities[1] - Classes of 1960/65, 1970, 1971, 1972 and 1973

1. Theoretical Development

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	58.0	51.0	51.0	54.3	51.3	52.5
Too Much	34.0	37.1	37.1	33.1	36.4	35.6
Too Little	8.0	10.5	8.2	8.2	9.6	9.0
No Opinion	0.0	1.4	3.7	4.4	2.6	2.9
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	100	210	245	317	343	1215

2. Useful Results

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	27.3	32.1	32.4	33.7	30.4	31.7
Too Much	1.0	1.4	1.7	1.0	0.9	1.2
Too Little	66.7	61.2	58.0	58.4	62.8	60.8
No opinion	5.0	5.3	8.0	7.0	5.9	6.4
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	99	209	238	315	339	1200

Source: Primary research by the authors. (Table 1, Question III.C)

3. Well Formulated Problems

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	54.5	55.8	61.1	61.7	57.9	58.9
Too Much	6.1	8.3	8.1	5.1	4.7	6.2
Too Little	26.3	26.2	25.2	23.8	32.4	27.1
No Opinion	13.1	9.7	5.6	9.3	5.0	7.7
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	206	234	317	340	1190

4. Unstructured Problems

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	23.7	26.7	28.7	34.4	30.6	30.0
Too Much	4.1	6.8	3.9	4.3	7.6	5.6
Too Little	49.5	39.8	47.0	39.7	36.4	41.0
No opinion	22.7	26.7	20.4	21.5	25.5	23.4
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	97	206	230	302	330	1165

5. Mathematical Modelling

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	13.4	20.6	25.4	26.2	26.5	24.1
Too Much	0.1	2.0	1.7	1.0	1.5	1.5
Too Little	71.1	54.4	51.3	51.7	46.4	52.2
No Opinion	14.4	23.0	21.6	21.2	25.6	22.3
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	97	204	232	302	332	1167

6. Applications to Other Fields

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	22.8	22.5	23.3	25.2	25.7	24.3
Too Much	0.0	0.5	0.4	1.3	1.2	0.8
Too Little	72.3	65.2	65.4	65.4	66.5	66.2
No Opinion	0.5	11.8	10.8	8.1	6.7	8.6
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	101	204	240	309	343	1197

7. History and Philosophy of Mathmeatics

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	31.3	31.6	29.7	29.2	29.4	29.9
Too Much	0.4	2.4	4.6	6.2	4.2	4.5
Too Little	50.5	49.0	46.4	45.8	46.1	47.0
No opinion	14.1	17.0	19.2	18.8	20.3	18.6
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	99	206	239	308	330	1182

8. Social Implications of Mathematics

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	13.7	11.7	12.8	11.2	15.5	13.0
Too Much	1.1	1.0	2.6	2.0	1.2	1.6
Too Little	56.8	65.5	59.6	62.5	60.5	61.4
No Opinion	28.4	21.8	25.1	24.3	22.8	24.0
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	95	206	235	304	343	1169

9. Lectures

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	68.9	59.3	59.1	57.9	66.9	61.9
Too Much	28.2	32.1	29.3	33.0	24.0	29.1
Too Little	1.9	3.8	4.5	3.6	4.7	4.0
No Opinion	1.0	4.8	7.0	5.5	4.4	5.0
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	103	209	242	309	338	1201

10. Seminars, Group Discussions

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	26.7	30.4	31.6	28.5	29.0	29.5
Too Much	2.0	2.0	2.5	1.6	2.1	2.0
Too Little	65.0	59.8	53.6	60.5	56.6	58.2
No opinion	6.9	7.8	12.2	9.4	12.3	10.3
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	101	204	237	309	334	1185

11. Individual Guidance

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	37.6	40.8	41.4	43.8	44.3	42.4
Too Much	2.0	0.0	0.8	0.6	0.9	0.8
Too Little	57.4	52.4	51.1	46.6	46.7	49.5
No Opinion	3.0	6.8	6.8	8.9	8.0	7.4
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	101	206	237	313	336	1193

12. Tutorials, Problem Sessions

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	44.4	51.5	52.5	56.4	51.9	52.5
Too Much	6.1	2.4	4.6	3.2	5.6	4.3
Too Little	46.5	41.3	36.7	32.7	33.5	36.3
No Opinion	3.0	4.9	6.3	7.7	8.9	6.9
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	206	240	312	337	1194

13. Independent Study

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	57.1	58.2	60.8	58.4	61.4	59.6
Too Much	13.3	4.5	7.2	5.8	6.2	6.6
Too Little	23.5	28.9	23.6	26.6	24.6	25.6
No opinion	6.1	8.5	8.4	9.1	7.7	8.2
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	98	201	237	308	337	1181

14. Original Work

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	38.8	32.2	31.6	26.2	30.4	30.6
Too Much	0.0	0.0	0.9	1.3	0.3	0.6
Too Little	45.9	48.3	47.4	51.3	53.6	50.2
No Opinion	15.3	19.5	20.1	21.2	15.7	18.6
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	98	205	234	302	332	1171

15. Developing Teaching Ability

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	18.8	22.8	19.0	19.5	19.1	19.8
Too Much	3.1	1.0	1.3	0.3	0.0	0.8
Too Little	53.1	52.4	56.5	51.8	50.9	52.7
No Opinion	25.0	23.8	23.3	28.4	30.0	26.7
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	96	206	232	303	330	1167

16. Written Assignments

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	74.0	76.3	70.0	74.8	71.4	73.0
Too Much	9.0	7.2	10.7	9.3	12.4	10.1
Too Little	14.0	10.6	14.8	11.2	11.2	12.1
No opinion	3.0	5.8	4.5	4.8	5.0	4.8
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	100	207	243	313	339	1202

17. Examinations

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	68.0	66.3	58.5	63.0	67.6	64.3
Too Much	29.4	29.3	34.4	30.1	25.7	29.5
Too Little	2.9	2.9	2.5	3.8	3.2	3.2
No Opinion	1.0	1.4	4.6	3.2	3.5	3.1
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of Respondents	102	208	241	316	339	1206

18. Teamwork

Year of Graduation	1960 & 1965	1970	1971	1972	1973	ALL
	%	%	%	%	%	
Opinions of emphasis placed						
About Right	14.7	27.0	24.8	26.1	24.5	24.6
Too Much	6.3	2.0	2.1	1.7	2.4	2.4
Too Little	56.8	56.4	54.3	55.8	57.9	56.3
No Opinion	22.1	14.7	18.8	16.5	15.2	16.7
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	204	234	303	335	1171

APPENDIX I

Table III.C.2 - Opinions regarding Content and Methodology of Graduate University Mathematical Education: All Classess, M.A.[1] - Mathematical Sciences - Canadian Universities

Content and Methodology	Opinions: About Right %	Too Much %	Too Little %	No Opinion %	Number of Res.
1. Theoretical development	71.7	19.0	6.8	2.5	353
2. Useful results	51.6	0.6	38.9	8.9	347
3. Well-formulated problems	60.3	4.7	22.7	12.2	343
4. Unstructured problems	44.2	5.3	27.0	23.4	337
5. Mathematical modelling	35.4	1.2	41.7	21.7	336
6. Applications to other fields	32.7	0.3	51.8	15.2	342
7. History and philosophy of math	27.3	1.2	47.5	24.0	336
8. Social Implications of math	19.2	0.3	53.3	27.2	338
9. Lectures	69.3	19.3	6.9	4.6	348
10. Seminars, group discussions	61.7	3.7	30.3	1.4	350
11. Individual guidance	61.8	0.0	32.5	5.7	348
12. Tuorials, problem sessions	57.5	1.5	27.1	13.9	339
13. Independent study	76.1	6.6	10.9	6.3	348
14. Original work	70.3	3.5	20.2	6.1	347
15. Developing teaching ability	39.5	1.5	44.8	14.2	344
16. Written assignments	73.0	10.3	12.4	4.3	348
17. Examinations	73.0	17.8	4.0	5.2	348
18. Teamwork	36.8	1.8	45.9	15.6	340

Source: Primary research by the authors. (Table 1, Question III.C)

Total - 100

APPENDIX I

Table III.C.3 - Opinions regarding Content and Methodology of Graduate University Mathematical Education: All Classes, Ph.D. - Mathematical Sciences - Canadian Universities [1]

	Opinions				
	About Right %	Too Much %	Too Little %	No Opinion %	Number of Res.
Content and Methodology					
. Theoretical development	75.6	15.6	6.9	1.9	160
2. Useful results	51.9	0.6	41.6	5.8	154
3. Well-formulated problems	62.7	6.0	18.7	12.7	150
4. Unstructured problems	33.6	0.7	40.9	24.8	149
5. Mathematical modelling	23.5	0.0	53.7	22.8	149
6. Applications to other fields	30.9	0.0	57.9	11.2	152
7. History and philosophy of math	26.8	0.0	56.4	16.8	149
8. Social Implications of math	17.6	0.7	50.7	31.1	148
9. Lectures	75.3	16.9	5.2	2.6	154
10. Seminars, group discussions	68.8	0.6	25.3	5.3	154
11. Individual guidance	69.9	0.6	25.3	3.2	154
12. Tuorials, problem sessions	57.0	0.0	22.8	20.2	149
13. Independent study	81.6	3.2	12.0	3.2	158
14. Original work	81.4	1.9	14.8	1.9	155
15. Developing teaching ability	43.8	1.3	48.4	6.5	153
16. Written assignments	76.8	3.2	16.1	3.9	155
17. Examinations	76.3	13.5	5.1	5.1	156
18. Teamwork	32.5	0.7	38.4	28.5	151

Source: Primary research by the authors. (Table 1, Question III.C)

Total - 100

APPENDIX I

Table I.I - Current Work Function, Industry by Degree Level: All Classes - Mathematical Sciences - Canadian Universities[1].

Level of last highest degree:	B.A. (N-1276) %	M.A. (N-386) %	PhD (N-168) %
Principal Function[1]			
Teaching and Training	22	37	78
Applied Mathematical Research	3	4	2
Basic Mathematical Research	1	2	5
Actuarial and Business Math	9	2	-
Operations Research	3	6	2
Experimental Design	1	2	1
Sampling, Surveying, Other Statistical Procedures	5	5	2
Systems Analysis, Simulation and Systems Eng., Other	21	19	2
Administration	4	2	1
Programming and Data Processing	5	3	1
Supervision (related to math)	-	1	-
Math Modelling, Consulting (related to math)	-	1	-
Other	9	2	-
No response, not applicable[2]	17	14	6
TOTAL	100	100	100
Industry			
Primary Industry	4	3	1
Manufacturing	5	4	1
Transportation and Communication	2	3	1
Public Utilities	2	2	1
Trade (Wholesale and Retail)	2	-	1
Finance and Insurance	11	4	-
Business, Service Industries	10	7	1
Federal Government	10	10	6
Provincial Government	5	4	-
Other Government	1	2	-
University	16	35	81
College (Post Secondary)	2	5	1
High School	10	5	-
Grade School	1	-	-
Other	3	1	1
No response, not applicable[2]	17	15	5
TOTAL	100	100	100

1 Respondents were allowed to specify up to three phrases describing their nature of work. The various possibilities were assigned priority numbers, and the highest priority used. The highest priority for those employed in the Academic Institutions was given to "Teaching and Training", the lowest to "Other". The highest priority for those employed elsewhere was given to "Applied Mathematical Research", the lowest to "Teaching and Training".

2 Includes graduates who are still students.

Source: Primary Research by the authors. (Table 1, Question I.I)

APPENDIX I

Table III.D.1.1 - Opinions Regarding the Degree of Consideration Given to Various Aspects of University Mathematical Education: All Classess,[1] B.Sc. (Hon.) - Mathematical Sciences - Canadian Universities

	Opinions				
	About Right %	Too Much %	Too Little %	No Opinion %	Number of Res. %
1. Origins of mathematical concepts and theories............	45.0	4.2	43.9	6.9	1,187
2. Interconnections between various branches of mathematics..	38.2	0.6	57.6	3.6	1,196
3. Limitations and abuses of mathematical methods.............	33.6	1.0	47.4	18.0	1,173
4. Communicating mathematical ideas to others.................	34.5	0.9	53.3	11.3	1,167
5. Developing computational skills.........................	64.4	4.2	25.8	5.6	1,185
6. Learning how to use the mathematical literature..............	30.1	0.4	60.7	8.8	1,187
7. Contact with users of mathematics (invited talks, etc.).....	31.5	0.3	60.5	7.8	1,186
8. Career opportunities in the mathematical sciences...........	35.7	0.7	55.7	7.9	1,175
9. The Impact of mathematics on modern society...................	22.1	1.0	61.6	15.3	1,165
10. Cultural aspects of mathematics.....................	15.6	1.0	56.6	26.9	1,154
11. Contemporary developments in mathematics.....................	37.2	0.8	51.4	10.7	1,181

Source: Primary research by the authors. (Table 1, Question II.D.1)

Total - 100

APPENDIX I

Table III.D.1.2 - Opinions Regarding the Degree of Consideration Given to Various Aspects of University Mathematical Education: Classes of 1960/65, 1970, 1971, 1972 and 1973 - Mathematical Sciences - Canadian Universities [1]

1. Origins of mathematical concepts and theories:

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Opinions re: Degree of consideration that was given:						
About right	41.4	45.8	37.3	47.4	48.9	45.0
Too much	3.0	3.8	5.8	4.3	3.6	4.2
Too little	41.7	43.4	48.5	41.4	41.7	43.9
No opinion	7.1	7.1	8.3	6.9	5.7	6.9
Number of respondents	99	212	241	304	331	1,187

2. Interconnections between various branches of mathematics:

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Opinions re: Degree of consideration that was given:						
About right	27.0	36.9	33.6	43.4	41.0	38.2
Too much	0.0	0.9	0.8	0.7	0.3	0.6
Too little	68.0	58.5	58.1	53.9	56.9	57.6
No opinion	5.0	3.7	7.5	2.0	1.8	3.6
Number of respondents	100	217	241	304	334	1,196

	3. Limitations and abuses of mathematical methods:					
Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	18.8	38.0	29.8	35.5	36.1	33.6
Too much	1.0	0.5	1.3	0.7	1.5	1.0
Too little	53.1	45.2	51.7	48.5	43.0	47.4
No opinion	27.1	16.3	17.2	15.3	19.4	18.0
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	96	208	238	301	330	1,173

	4. Communicating mathematical ideas to others					
Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	25.5	34.9	27.3	35.1	41.7	34.5
Too much	1.0	0.5	1.3	1.7	0.0	0.9
Too little	54.1	53.6	60.1	53.0	48.2	53.3
No opinion	19.4	11.0	11.3	10.1	10.1	11.3
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	98	209	238	296	326	1,167

5. Developing computational skills

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	61.6	60.2	61.7	69.4	65.3	64.4
Too much	2.0	4.3	4.2	3.6	5.4	4.2
Too little	31.3	28.9	27.5	22.0	24.5	25.8
No opinion	5.1	6.6	6.7	4.9	4.8	5.6
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	211	240	304	331	1,185

6. Learning how to use mathematical literature

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	28.6	31.0	30.2	31.5	28.6	30.1
Too much	0.0	0.5	0.8	0.7	0.0	0.4
Too little	61.2	60.5	59.9	60.7	61.1	60.7
No opinion	10.2	8.1	9.1	7.2	10.2	8.8
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	98	210	242	305	332	1,187

7. Contact with users of mathematics (invited talks, etc.)

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	25.3	26.8	29.8	32.3	36.6	31.5
Too much	0.0	0.0	0.4	0.0	0.6	0.3
Too little	66.7	65.1	62.8	62.0	53.0	60.5
No opinion	8.0	8.1	7.0	5.7	9.8	7.8
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	99	209	242	300	336	1,186

8. Career opportunities in the mathematical sciences

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	23.7	36.1	31.5	38.5	39.3	35.7
Too much	0.0	0.5	0.8	1.3	0.3	0.7
Too little	60.8	55.3	59.7	53.5	53.8	55.7
No opinion	15.5	8.2	8.0	6.6	6.6	7.9
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	97	208	238	301	331	1,175

9. The impact of mathematics on modern society

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	18.9	20.1	17.4	23.9	25.9	22.1
Too much	1.1	0.5	1.3	1.7	0.6	1.0
Too little	63.2	62.2	64.4	61.4	59.0	61.6
No opinion	16.8	17.2	16.9	13.0	14.5	15.3
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	209	236	2930	332	1,165

10. Cultural aspects of mathematics

Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	21.1	15.0	14.8	14.8	15.6	15.6
Too much	1.1	1.0	0.8	1.0	0.9	1.0
Too little	49.5	57.8	54.2	58.1	58.3	56.6
No opinion	28.3	26.2	30.1	26.1	25.2	26.9
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	95	206	236	291	326	1,154

	11. Contemporary Developments in Mathematics					
Year of Graduation:	1960/65 %	1970 %	1971 %	1972 %	1973 %	ALL %
Degree of consideration given						
About right	24.0	38.0	32.9	39.9	41.3	37.2
Too much	1.0	0.5	0.4	1.0	0.9	0.8
Too little	61.0	51.9	52.9	50.5	47.9	51.4
No opinion	14.0	9.6	13.8	8.6	9.9	10.7
Total	100.0	100.0	100.0	100.0	100.0	100.0
Number of respondents	100	208	240	301	332	1,181

APPENDIX I

Table III.D.2 - Opinions regarding the Degree of Consideration given to Various Aspects of Graduate University Mathematical Education: All Classes, M.A. - Mathematical Sciences - Canadian Universities[1]

	Opinions				
	About Right %	Too Much %	Too Little %	No Opinion %	Number of Res.
1. Origins of mathematical concepts and theories............	52.2	2.0	40.1	5.8	347
2. Interconnections between various branches of mathematics..	47.9	0.3	45.0	6.9	349
3. Limitations and abuses of mathematical methods.............	46.3	1.2	36.4	16.1	341
4. Communicating mathematical ideas to others..................	46.9	1.2	42.0	9.9	343
5. Developing computational skills..........................	66.7	2.0	21.9	9.4	342
6. Learning how to use the mathematical literature...............	63.6	0.9	30.6	4.9	346
7. Contact with users of mathematics (invited talks, etc.).....	62.2	0.9	32.8	4.1	344
8. Career opportunities in the mathematical sciences............	40.8	1.5	47.8	9.9	343
9. The Impact of mathematics on modern society...................	30.3	2.4	54.1	13.2	340
10. Cultural aspects of mathematics......................	20.1	1.5	53.8	24.6	333
11. Contemporary developments in mathematics......................	66.5	2.0	24.8	6.7	343

Source: Primary research by the authors (Table 1, Question III.D)

Total - 100%

APPENDIX I

Table III.D.3 - Opinions Regarding the Degree of Consideration given to Various Aspects of Graduate University Mathematical Education: All Classes, Ph.D. - Mathematical Sciences - Canadian Universities [1]

	Opinions				
	About Right %	Too Much %	Too Little %	No Opinion %	Number of Res.
1. Origins of mathematical concepts and theories............	46.2	0.6	51.3	1.9	156
2. Interconnections between various branches of mathematics..	44.8	1.3	49.4	4.5	154
3. Limitations and abuses of mathematical methods.............	46.1	0.0	36.2	17.8	152
4. Communicating mathematical ideas to others..................	52.3	0.0	39.7	7.9	151
5. Developing computational skills..........................	66.7	1.3	24.2	7.8	153
6. Learning how to use the mathematical literature...............	69.9	0.0	26.1	3.9	153
7. Contact with users of mathematics (invited talks, etc.).....	67.5	0.6	26.2	5.7	157
8. Career opportunities in the mathematical sciences............	46.7	0.0	45.4	7.9	152
9. The Impact of mathematics on modern society..................	26.0	0.0	58.0	16.0	150
10. Cultural aspects of mathematics......................	25.8	0.0	50.3	23.8	151
11. Contemporary developments in mathematics.....................	72.1	1.3	23.4	3.2	154

Source: Primary research by the authors (Table 1, Question III.D)

Total - 100%

APPENDIX II

We had hoped to be able to show some of the comparative findings of the Highly Qualified Manpower Survey. Unfortunately, we cannot until the pending series of Reliability Reports from Statistics Canada have been accepted by the Ministry of State for Science and Technology.

It is our hope that the data will be made available in time for our final edition.

APPENDIX III
GRADUATION PATTERNS AND FACULTY CHARACTERISTICS

The accompanying tables provide some data on graduation patterns in the Mathematical Sciences at the Bachelor's, Master's and Doctor's levels as well as faculty characteristics. For the purposes of the Mathematical Study, the Mathematical Sciences include Pure Mathematics, Statistics, Actuarial Science, and Computer Science. These figures have been compiled from information supplied by the following Canadian University Departments.

ACADIA UNIVERSITY - Computer Science
UNIVERSITY OF Alberta - Mathematics
ALTHOUSE COLLEGE OF EDUCATION (U. of Western Ontario) - Mathematics (Education)
BISHOP'S UNIVERSITY - Mathematics
UNIVERSITY OF BRITISH COLUMBIA - Computer Science
BROCK UNIVERSITY - Mathematics
UNIVERSITY OF CALGARY - Mathematics
DALHOUSIE UNIVERSITY - Mathematics
UNIVERSITY OF GUELPH - Mathematics and Statistics
LAKEHEAD UNIVERSITY - Mathematics
UNIVERSITE LAVAL - Mathematics, Computer Science
UNIVERSITY OF LETHBRIDGE - Mathematical Sciences
LOYOLA COLLEGE - Computer Science
UNIVERSITY OF MANITOBA - Mathematics, Statistics
UNIVERSITY OF McGILL - Mathematics
MEMORIAL UNIVERSITY OF NEWFOUNDLAND - Mathematics
UNIVERSITE DE MONCTON - Mathematics
UNIVERSITY OF NEW BRUNSWICK - Mathematics
OTTAWA UNIVERSITY - Mathematics
UNIVERSITY OF PRINCE EDWARD ISLAND - Mathematics
UNIVERSITE DU QUEBEC a Chicoutimi - Mathematics
a Montreal - Mathematics
a Trois Rivieres - Mathematics
QUEEN'S UNIVERSITY - Mathematics, Computing and Information Science
ROYAL MILITARY COLLEGE OF CANADA - Mathematics
UNIVERSITY OF SASKATCHEWAN - (Regina) - Mathematics
(Saskatoon) - Mathematics, Computer Science
UNIVERSITE DE SHERBROOKE - Mathematics
SIMON FRASER UNIVERSITY - Mathematics
SIR GEORGE WILLIAMS UNIVERSITY - Mathematics
UNIVERSITY OF TORONTO - Mathematics, Computer Science
UNIVERSITY OF VICTORIA - Mathematics
UNIVERSITY OF WATERLOO - Faculty of Mathematics

UNIVERSITY OF WESTERN ONTARIO - Mathematics, Applied Mathematics and Computer Science
UNIVERSITY OF WINDSOR - Mathematics and Computer Science
UNIVERSITY OF WINNIPEG - Mathematics and Statistics
YORK UNIVERSITY - Mathematics

APPENDIX IV

Additional Information on Graduation Patterns and Faculty Characteristics

Our enquiry into the graduation patterns and faculty characteristics of Appendix III can to some extent, be compared to statistics gathered through Statistics Canada, the Annual Statistical Reports of the Canadian Association of Graduate Schools, and the work done by K.D. Hunt of the University of Waterloo. Tables 1 and 2 report on the number of doctoral degrees awarded in the mathematical sciences. Table 1 considers the calendar year, whereas Table 2 considers the school year as defined by Statistics Canada. Table 3 shows the number and doctoral qualifications of mathematics faculty members at Canadian Universities by school year.

Table 1 - Number of Doctorates awarded in Mathematics by Canadian Universities, 1961 - 1973.

Table 1

Year	Number of PhD's Awarded
1961	11
1962	9
1963	15
1964	19
1965	42
1966	39
1967	52
1968	49
1969	60
1970	63
1971	86
1972	88
1973	94

Note: For the years 1961 to 1968 the figures are based on a list produced by K.D. Hunt of the University of Waterloo of the names and thesis titles of PhD's in mathematics for 1961-1972. For the years 1969-1973 the figures quoted are from the annual Statistical Reports of the Canadian Association of Graduate Schools.

Table 2

NUMBER OF GRADUATE DEGREES AWARDED IN MATHEMATICS BY CANADIAN UNIVERSITIES, 1960/61 - 1971/72

YEAR	MASTERS NUMBER	MASTERS INDEX	DOCTORATES NUMBER	DOCTORATES INDEX
1960-61	55	100	8	100
1961-62	46	84	10	125
1962-63	82	149	06	75
1963-64	106	193	21	263
1964-65	83	151	28	350
1965-66	171	311	34	425
1966-67	175	318	43	538
1967-68	213	387	49	613
1968-69	247	449	53	663
1969-70	392	713	61	763
1970-71	357	649	85	1,063
1971-72	428	778	97	1,213

Source: Derived from Statistics Canada Data.

Table 3

NUMBER AND DOCTORAL QUALIFICATIONS OF MATHEMATICS FACULTY AT CANADIAN UNIVERSITIES, 1956/57 - 1973/74

YEAR	NUMBER	PERCENTAGE WITH DOCTORATE
1956-57	237	49.8
1958-59	311	48.9
1960-61	388	49.2
1962-63	458	50.3
1963-64	545	47.3
1965-66	731	51.2
1967-68	962	54.1
1968-69	1,038	60.5
1969-70	1,200	62.3
1970-71	1,269	67.5
1971-72	1,295	69.5
1972-73	1,122*	76.6
1973-74	1,085*	78.2

*unadjusted

Source: Adapted from Statistics Canada.

APPENDIX V

NRC OPERATING GRANTS 1970/74

	All		Biology		Chemistry		Physics		Comp. Sc.		Mathematics	
	$	Av.	$	Av.	$	Av.	$	Av.	$	Av.	$	Av.
1970:	34.4	8.2	8.0	8.3	6.2	10.4	3.3	8.5	0.8	6.9	2.1	4.1
1971:	36.2	8.0	8.5	8.1	6.0	10.4	3.6	8.7	0.9	6.6	2.2	3.8
1972:	37.6	7.8	9.0	8.2	6.0	10.4	3.7	8.4	1.0	6.0	2.2	3.5
1973:	40.1	8.2	9.4	8.4	6.4	11.8	3.6	8.9	1.2	6.9	2.4	3.6
1974:	42.2	8.4	10.1	8.6	6.5	12.1	3.8	9.0	1.3	6.7	2.4	3.5

Note: The column headed $ gives the total awards in millions of dollars (excludes equipment).
The Column headed Av. gives the average award per grantee in thousands of dollars.
The first two columns, All, records the awards given by the NRC on the advice of its 16 Discipline Grant Selection Committees.
Between 1970 and 1974 the number of Operating Grants awarded by the NRC increased from 4261 to 5067; in Mathematics from 505 to 707; and in Computing Science from 113 to 196.

Remarks: The combined total for Mathematics and Computer Science each year was less than for any of the other committees reported above.
The average grant given to Mathematics was less than 50% of the overall average grant.
The overall average grant has not increased appreciably in five years so that there has been a decrease of 25% or more in support to individual researchers measured in constant dollars.

APPENDIX VI

NRC Operating Grants to Mathematics and Computing Science
(from the first in 1958 to 1974)

Year	Mathematics	Computing
1958	50.0	
1959	81.0	
1960	87.5	
1961	120.0	
1962	158.5	
1963	181.0	
1964	306.0	
1965	446.0	
1966	724.0	268
1967	1,353.0	302
1968	1,788.0	462
1969	2,115.0	599
1970	2,076.0	796
1971	2,244.0	874
1972	2,217.0	968
1973	2,403.0	1,236
1974	2,446.0	1,304

Note:
- The above figures are in thousands of dollars.
- Some money was granted for Computing in 1963 and possibly earlier but was not listed separately.
- Travel Fellowships, which in 1971, amounted to 19,000 and Special Computing Grants to $118,000 are not included in these figures. The latter type of grant has been phased out.
- Overall in the NRC Budget the ratio of Operating Grants to individuals versus Block Grants (including Negotiated Development Grants) has been approximately 10:1.

APPENDIX VII

Post Doctoral Fellows and Research Associates in Mathematics

The following number of PDF's and RA's were abstracted from returns of a questionnaire sent to Canadian University Mathematics Departments. Apparently diverse interpretations were given by the respondents to the questions so the meaning of the following figures is in doubt. Financial data which accompanied them suggest that the average annual payment to these individuals was approximately $6,000 (in some cases, this was for nine or ten months only).

Year	Pure	Applied	Statistics	Computing	Total
69/70	67	34	12	3	116
70/71	70	36	7	9	122
71/72	64	35	14	2	115
72/73	90	30	23	10	153
73/74	80	38	16	7	141

APPENDIX VIII

DECEMBER, 1973

DISTRIBUTION OF N.R.C. OPERATING GRANTS IN THE MATHEMATICAL SCIENCES FOR 1973/74

The list below analyzes NRC Operating Grants from 1973/74 according to the Field Code for which the grant was requested. Most of these requests were dealt with by the NRC Awards Committee 107 or 116. However a few were referred to other Committees in the following list: 104 (Chemical/Metallurgical Engineering); 106 (Civil Engineering); 107 (Computing/Information Science); 110 (Earth Sciences); 111 (Electrical Engineering); 112 (Psychology); 113 (Mechanical Engineering); 115 (Physics); 116 (Pure/Applied Mathematics); 120 (Industrial Engineering); 121 (Interdisciplinary).

FIELD		NUMBER OF		AMOUNT		
CODE	DESCRIPTION	APPLICANTS	AWARDS	REQUESTED	GRANTED	GRAND TOTAL
0001	Logic and Foundations	26	22	144,274	74,023	74,023
0003	Set Theory	4	4	52,970	27,300	27,300
	Committee 116	3	3	42,550	23,300	
	Committee 107	1	1	10,420	4,000	
0004	Combinatorial Theory, Graph Theory	38	37	465,315	182,212	182,212
	Committee 116	37	36	446,735	174,712	
	Committee 107	1	1	18,580	7,500	
0005	Order, Lattices, Ordered Algebraic Structures	9	8	158,760	60,700	60,700
0006	General Mathematical Systems	12	11	138,237	60,604	60,604
0007	Theory of Numbers	28	26	187,410	79,539	79,539
0008	Fields and Polynomials	2	2	35,620	23,600	23,600
0009	Commutative Associative Rings and Algebras	7	7	33,740	12,400	12,400
0010	Linear and Multilinear Algebra, Matrix Theory	11	11	86,215	39,500	39,500

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	NUMBER OF AWARDS	AMOUNT REQUESTED	AMOUNT GRANTED	GRAND TOTAL
0011	Associative Rings and Algebras	19	18	142,660	60,262	60,262
0012	Nonassociative Rings and Algebras	7	7	59,140	21,900	21,900
0013	Category Theory	13	13	94,480	18,000	18,000
0014	Homological Algebra	5	5	35,100	6,900	6,900
0015	Group Theory and Generalizations	25	20	212,855	86,769	86,769
0016	Topological Groups and Lie Theory	4	3	10,660	2,500	2,500
0017	Functions of Real Variables	3	3	23,300	9,166	9,166
0018	Measure and Integration	14	14	146,750	73,189	73,189
0019	Functions of a Complex Variable	10	10	95,077	42,050	42,050
0020	Potential Theory	5	5	46,045	19,000	19,000
0021	Several Complex Variables	3	3	6,800	4,700	4,700
0022	Special Functions	8	8	84,507	33,967	33,967
	Committee 116	6	6	63,537	27,467	
	Committee 107	2	2	20,970	6,500	
0023	Ordinary Differential Equations	26	25	225,105	75,321	75,321
	Committee 116	25	24	213,435	72,321	
	Committee 107	1	1	11,670	3,000	
0024	Partial Differential Equations	26	22	186,290	95,904	95,904
0025	Finite Differences and Functional Equations	6	6	68,968	34,470	34,470
0026	Sequences, Series, Summability	5	5	51,150	31,700	31,700

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	AWARDS	AMOUNT REQUESTED	GRANTED	GRAND TOTAL
0027	Approximations and Expansions	12	10	90,780	47,658	47,658
0028	Fourier Analysis	10	9	75,041	36,338	36,338
0029	Integral Transforms, Operational Calculus	4	4	30,636	14,500	14,500
0030	Integral Equations	4	4	34,300	14,000	14,000
0031	Functional Analysis	46	39	331,750	111,434	111,434
	Committee 116	44	37	316,050	104,434	
	Committee 111	2	2	15,700	7,000	
0032	Operator Theory	17	16	149,600	53,640	53,640
0033	Calculus of Variations, Optimal Control	5	4	52,100	13,653	13,653
0034	Geometry	21	19	164,875	84,680	84,680
0035	Convex Sets and Geometric Inequalities	3	3	17,400	9,500	9,500
0036	Differential Geometry	15	14	98,550	27,952	27,952
0037	General Topology	23	22	129,797	47,209	47,209
0038	Algebraic Topology	19	17	120,540	44,952	44,952
0039	Topology and Geometry of Manifolds	19	18	116,560	36,288	36,288
0040	Probability Theory	17	15	180,960	69,329	69,329
	Committee 116	16	14	168,610	65,829	
	Committee 107	1	1	12,350	3,500	
0041	Stochastic Processes, General	8	7	63,525	29,045	29,045
0042	Markov Processes	5	5	31,840	15,704	15,704
0043	Applied Probability	3	3	25,540	2,500	2,500

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	NUMBER OF AWARDS	AMOUNT REQUESTED	AMOUNT GRANTED	GRAND TOTAL
0044	Mathematical Statistics	16	14	145,167	41,170	41,170
0045	Decision Theory	6	6	58,295	27,758	27,758
0046	Distribution - Free and Non Parametric Methods	8	7	66,690	21,270	21,270
0047	Distribution Theory	4	4	46,400	12,606	12,606
0048	Foundations of Statistical Inference	5	5	82,399	43,021	43,021
0049	Multivariate Analysis	9	8	99,260	30,386	30,386
0050	Techniques of Statistical Inference; Estimation Testing, Sequential Procedures	20	17	182,216	64,771	64,771
	Committee 116	19	17	174,362	64,771	
	Committee 107	1	0	7,854	-	
0051	Monte Carlo Methods	3	3	24,910	8,537	8,537
0052	Data Analysis	4	4	50,800	15,551	15,551
	Committee 116	2	2	12,820	1,551	
	Committees 106, 112	2	2	37,980	14,000	
0053	Design and Analysis of Experiments	9	9	93,944	19,655	19,655
0054	Sampling	3	2	13,250	2,601	2,601
0055	Time Series Analysis	8	6	28,770	7,301	7,301
0056	Applications (biometrics, psychometrics, econometrics, sociometrics, engineering statistics, etc.)	14	10	137,242	52,809	52,809
	Committee 116	10	9	102,662	47,609	
	Committees 104, 107	4	3	34,580	5,200	

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	NUMBER OF AWARDS	AMOUNT REQUESTED	AMOUNT GRANTED	GRAND TOTAL
0057	Numerical Analysis	14	12	81,431	36,104	36,104
	Committee 116	12	10	69,021	27,604	
	Committee 107	2	2	12,410	8,500	
0058	Mechanics of Particles and Systems					
	Committee 115	1	1	6,880	3,500	3,500
0059	Elasticity, Plasticity	7	6	36,380	12,804	12,804
0060	Fluid Mechanics, Acoustics	12	10	153,580	55,552	55,552
	Committee 116	9	8	120,720	44,052	
	Committees 104, 113	3	2	32,860	11,500	
0061	Optics, Electromagnetic Theory	3	3	19,440	2,850	2,850
	Committee 116	2	2	15,320	2,100	
	Committee 115	1	1	4,120	750	
0063	Quantum Mechanics	4	4	59,429	18,700	18,700
0064	Statistical Physics, Structure of Matter	2	2	9,870	2,200	2,200
0065	Relativity	15	14	107,880	37,558	37,558
	Committee 116	14	13	98,880	35,558	
	Committee 115	1	1	9,000	2,000	
0067	Systems, Control	13	13	87,643	38,968	38,968
	Committee 116	10	10	59,143	17,002	
	Committees 107, 111	3	3	28,500	21,966	
0068	Information and Communications, Circuits	5	3	43,290	15,100	15,100
	Committee 116	3	2	28,500	10,900	
	Committees 107, 111	2	1	14,790	4,200	
0069	Miscellaneous (unspecified)	14	13	197,218	79,569	79,569
	Committee 116	12	11	155,008	58,569	
	Committees 107, 113	2	2	42,210	21,000	

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	AWARDS	AMOUNT REQUESTED	GRANTED	GRAND TOTAL
0200	Theory of Computation	12	10	130,303	73,600	73,600
	Committee 107	11	9	124,153	57,600	
	Committee 116	1	1	6,150	1,600	
0201	Automata Theory	6	5	82,510	57,250	57,250
	Committee 107	5	4	70,090	44,250	
	Committee 111	1	1	12,420	13,000	
0202	Artificial Intelligence	11	9	192,954	69,000	69,000
	Committee 107	10	8	155,454	41,000	
	Committee 111	1	1	37,500	28,000	
0203	Pattern Recognition	12	8	175,918	52,500	52,500
	Committee 107	8	5	137,078	34,000	
	Committees 110, 115, 121	4	3	38,840	18,500	
0204	Analysis of Algorithms	2	2	23,138	16,500	16,500
0205	Numerical Mathematics	34	34	433,619	225,603	225,603
	Committee 107	31	31	411,169	220,500	
	Committee 116	3	3	22,450	5,103	
0206	Computer-Related Discrete Maths	5	5	131,450	50,503	50,503
	Committee 107	3	3	102,270	48,000	
	Committee 116	2	2	29,180	2,503	
0207	Computer-Related Combinatorial Maths	7	4	107,247	24,500	24,500
0208	Computational Linguistics	2	2	14,080	5,500	5,500
0220	Programming Language	5	5	104,520	23,000	23,000
0221	Programming Language Processors	14	13	251,553	73,000	73,000

FIELD CODE	DESCRIPTION	NUMBER OF		AMOUNT		
		APPLICANTS	AWARDS	REQUESTED	GRANTED	GRAND TOTAL
0222	Data Structures	3	2	56,919	36,000	36,000
0223	Operating Systems	6	6	163,273	29,000	29,000
0224	Real-Time Software Systems	3	2	58,716	9,000	9,000
0225	Software Engineering	7	7	208,601	64,500	64,500
0230	Computer Organization	5	5	66,515	20,000	20,000
0231	Computer Systems Studies	11	11	166,118	72,500	72,500
0232	Computer Communications	1	1	14,600	11,000	11,000
0233	Computer Graphics	6	4	201,932	22,500	22,500
	Committee 107	5	3	199,542	22,000	
	Committee 116	1	1	2,390	500	
0235	Computing Aids to Design	4	2	30,133	9,000	9,000
	Committee 107	3	2	23,733	9,000	
	Committee 113	1	0	6,400	-	
0236	Computing Aids to Instruction	2	0	10,600	-	-
0238	Other Computing Science (unspecified)	4	4	134,835	19,000	19,00
	Committee 107	3	3	85,035	15,500	
	Committee 111	1	1	49,800	3,500	
0250	Data Processing	3	1	31,660	3,500	3,500
	Committee 107	2	1	21,740	3,500	
	Committee 103	1	0	9,920	-	
0251	File Organization and Management	1	1	42,178	20,000	20,000
0252	Information Storage and Retrieval	6	5	86,349	40,500	40,500
	Committee 107	4	3	58,204	33,000	
	Committees 111, 116	2	2	28,145	7,500	

FIELD CODE	DESCRIPTION	NUMBER OF APPLICANTS	NUMBER OF AWARDS	AMOUNT REQUESTED	AMOUNT GRANTED	GRAND TOTAL
0253	Automatic Information Analysis and Indexing	1	1	8,400	6,000	6,000
0254	Natural Language Processing	1	0	51,030	-	-
0255	Information Systems	3	2	42,320	9,000	9,000
0270	Mathematical Programming	22	17	263,565	106,003	106,003
	Committee 107	16	12	154,460	57,250	
	Committees 116, 120	6	5	109,105	48,753	
0271	Network Analysis and Flows	2	2	17,100	7,000	7,000
	Committee 107	1	1	9,000	2,500	
	Committee 113	1	1	8,100	4,500	
0272	Inventory Analysis	1	1	15,440	8,000	8,000
0273	Queueing Theory	5	5	58,760	24,704	24,704
	Committee 107	2	2	26,100	9,500	
	Committees 116, 120	3	3	32,660	15,204	
0274	Modelling and Simulation	7	4	100,212	40,500	40,500
	Committee 107	5	4	89,192	40,500	
	Committees 111, 120	2	0	11,020	-	
0275	Scheduling Theory	2	2	32,430	10,500	10,500
0276	Theory of Games	3	2	44,100	7,600	7,600
	Committee 107	2	1	36,190	6,000	
	Committee 116	1	1	7,910	1,600	
0277	Statistical Methods in Operations Research	3	3	54,580	36,000	36,000
0279	Other Operations Research (unspecified)	4	3	45,813	26,500	26,500
	Committee 107	2	2	15,800	14,500	
	Committees 110, 120	2	1	30,013	12,000	
				9,991,071	3,793,662	3,793,662

APPENDIX IX

JOB FORECASTS OF CANADIAN MATHEMATICS GRADUATES FOR 1974-1976
Provided by Otto Tomasek, Based on the Mathematics Study Questionnaire

Objective

To forecast the number of Canadian mathematics graduates (bachelors, masters and doctorates) by nature of work and category of employer for 1974-1976.

Data

1. Job history of Canadian mathematics graduates from survey results for the years 1960, 1965, 1970, 1971, 1972 and 1973. The survey results for the doctorate graduates were totaled for the 1960-64 period.

2. The total number of Canadian mathematics graduates for the 1960, 1965, 1970, 1971, 1972 and 1973.

Assumptions

1. The results of the surveys are representative of the population.

2. The total numbers of Canadian mathematics graduates are exact.

3. Only graduate's first job will be forecasted. Thus the total number of graduates in the labour force for each forecasted year will be equal to the forecasted number of graduates for this year.

Forecasting Techniques

1. Use simple linear regression to forecast the number of mathematics bachelors, masters and doctorates in 1974 - 1976. Time is used as independent variable and the number of graduates as dependent variable. The average number of doctorate graduates was used for the periods 1960-64 and 1965-69.

2. Use simple linear regression to forecast the percentage of mathematics graduates by nature of work and category of employer. Time is used as independent variable and percentages given by the survey as dependent variables. Adjust percentages so that they add up to 100% for each forecasted year.

3. Apply forecasted percentages (found in #2) to forecasted number of graduates (found in #1).

Results

1. The total number of mathematics graduates were all significant at the 95% level for the year 1974 - 1976.

2. Most of the forecasted numbers of graduates within each category were not significant at either the 90% or the 95% level.

3. Job forecasts are shown in the attached tables. In each table, the numbers in parentheses are row percentages rounded off to the nearest integer. The first line of each table "Significant Level" indicates whether the forecasts are significantt at 90%, 95% level or not significant (NS).

APPENDIX X
EMPLOYMENT OF MATHEMATICS PH.D'S IN CANADA

About sixty per cent of the 19772-73 graduates obtained positions in Canadian teaching institutions (universities or colleges), while another quarter went outside the country to obtain positions.

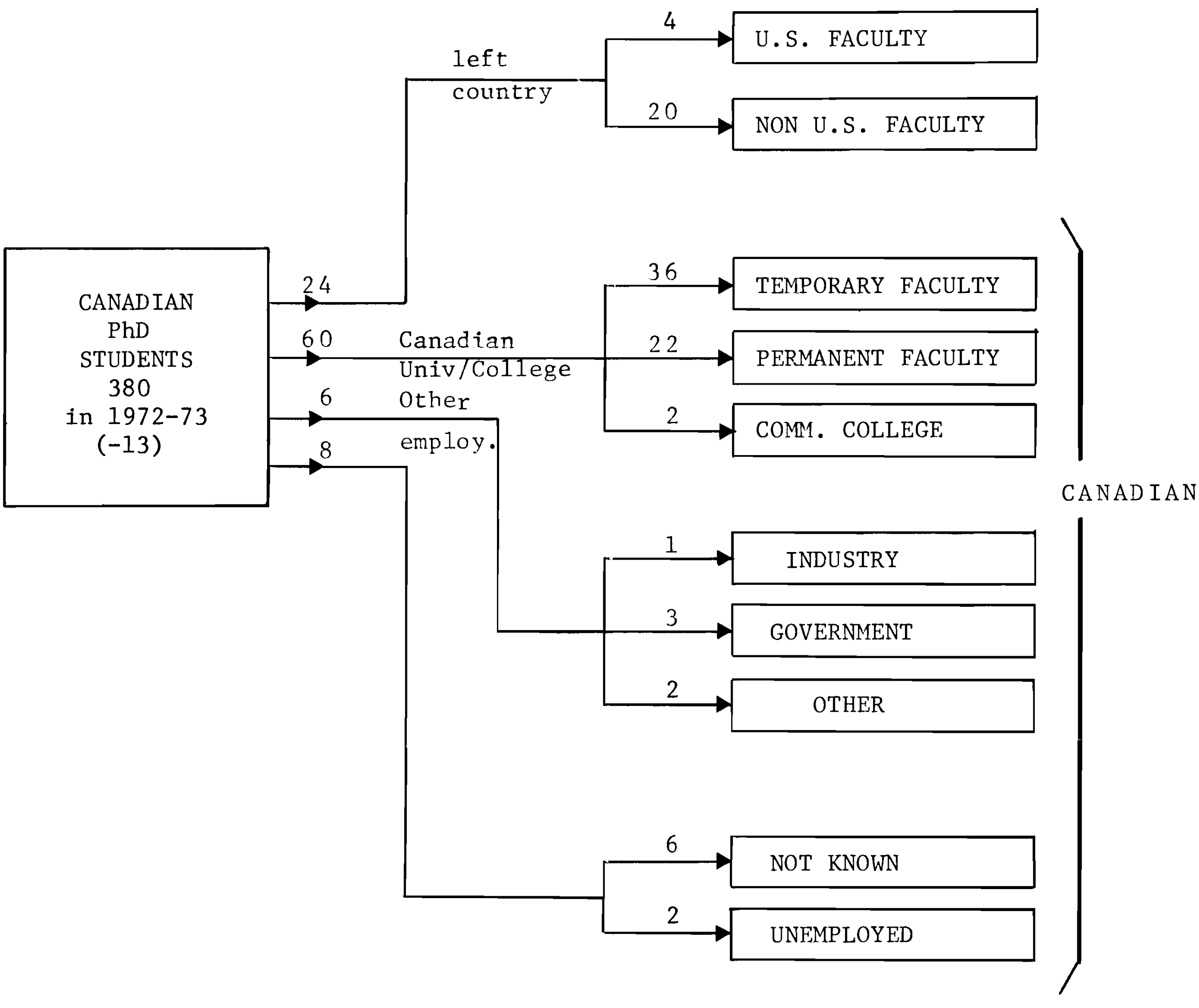


Figure 1 NEW CANADIAN MATHEMATICS PhD's (1972-73)

The population of Ph.D. candidates declined somewhat: 84 students entered programs in 1972-73, while 98 were graduated. The number of faculty positions in Mathematics in Canadian universities increased from 847 to 874.

Canadian universities hired 50 new Canadian Ph.D.'s. With an increase of only 27 staff positions total (and also competition from American and foreign students), it is obvious that net migration to other countries or non-teaching occupations occurred. The various "flows" are identified in Figure #2.

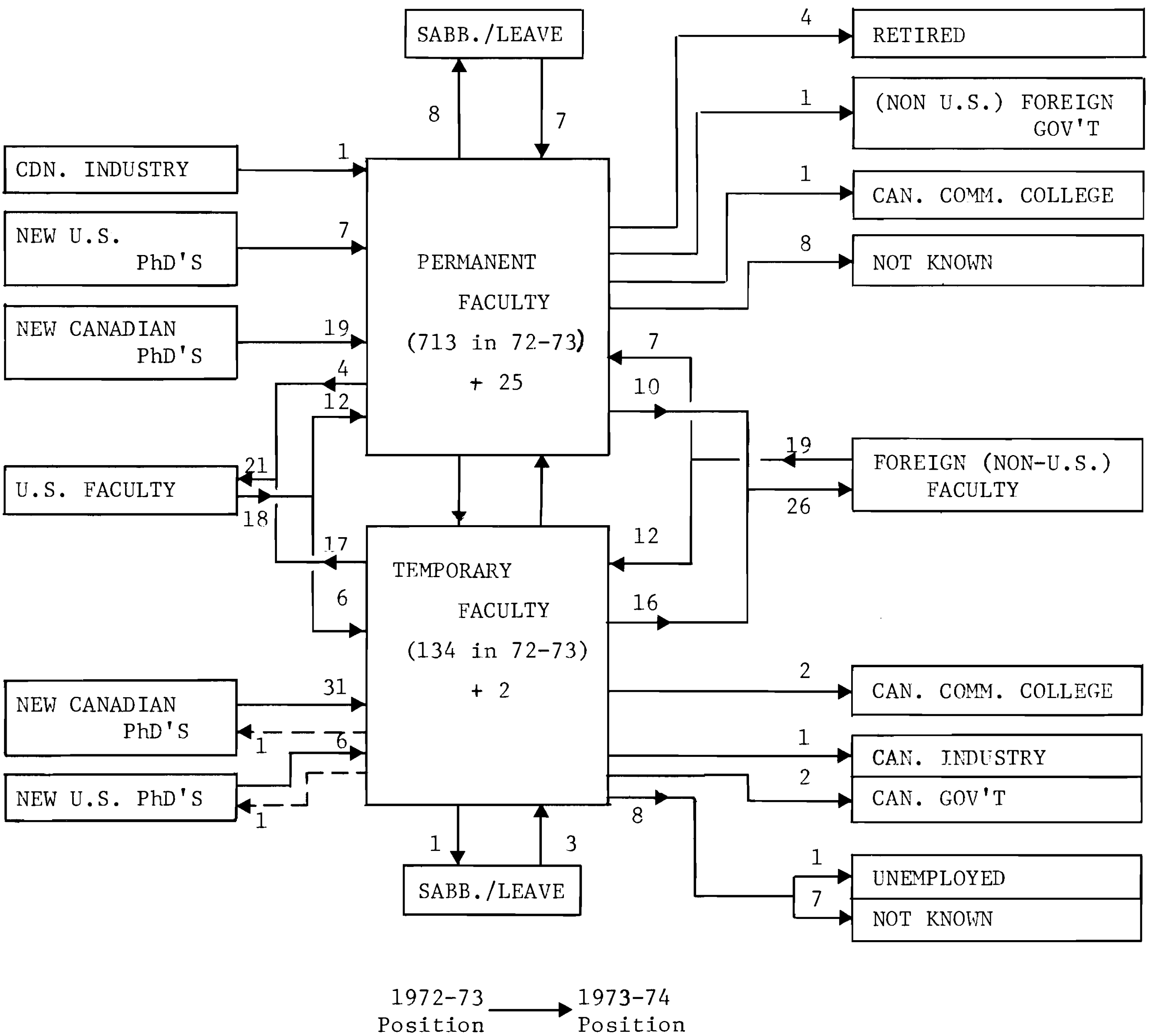
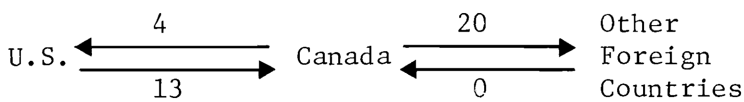


Figure 2 CANADIAN MATHEMATICS FACULTY

Faculty changes resulted in a net migration out of Canada, both to the U.S. and to other foreign countries. Of the 63 positions given to new graduates, 50 (79%) were filled by Canadians.

Twenty-four Canadian Ph.D.'s obtained positions outside Canada, of which only 4 were in the United States. Conversely, American graduates received 13 positions in Canada, while no non-American foreign new Ph.D.'s received Canadian positions.



The following table indicates percentages for the country of graduation for different faculty positions:

Position	Canadian	American	Other	N
Professor	25.0	35.6	39.4	180
Professor and Chairman	36.4	54.5	9.1	11
Associate Professor	33.1	37.6	29.3	242
Assistant Professor	43.7	44.9	11.4	242
Post-Doctoral	54.8	21.4	23.8	42

The percentages listed are for 1973-74. In the previous year, 40.7% of assistant professors and only 22.2% of post-doctorals were from Canadian institutions. In 1972, 4 of 18 visiting assistant professors were from Canadian Ph.D. programs, while in 1973, 11 of 23 were from Canada.

While slightly more faculty members received their Ph.D.'s from the U.S. than Canada, this is not true for recent Ph.D.'s. For faculty members who received their Ph.D.'s between 1970 and 1973, 165 received their degrees in Canada, 99 in the United States, and 25 outside Canada and the U.S. For all the major faculty positions in 1973-74 (Professor, Associate, Assistant, Lecturer, Post-Doctoral, Visiting Assistant) of those faculty members who graduated after 1969, the majority graduated from Canadian institutions.

The year of graduation for Canadian faculty broke down as follows:

Table 1: Year of Graduation of Canadian Faculty

Year of Graduation	N	Percentage
1970-73	301	32.8
1965-69	316	34.4
1960-64	137	14.9
1955-59	67	7.3
1950-54	46	4.9
1945-49	20	2.2
1940-44	10	1.1
1935-39	10	1.1
1930-34	9	0.9
1925-29	3	0.3

If an average "teaching life" of twenty-five or thirty years is assumed, only a small percentage of the population will approach the retirement point in the near future; the 1973 retirement figure of 4 people is probably representative of how large this figure will be for some years to come.

These statistics have been obtained by Douglas Baer and Professors Sydney Bulman-Fleming, Henry Crapo, and Edward Wang at the University of Waterloo and Wilfrid Laurier University. A copy of their complete report, five pages in length, and containing the usual warnings about incomplete data, has been mailed to the Chairman of each department participating in the survey. Additional copies may be requested from the Department of Pure Mathematics, University of Waterloo.

APPENDIX XI

Contributions to the Study

Here we list the names of those individuals who helped the Mathematics Study by writing letters or briefs concerning some aspect of the situation of mathematics in Canada. All of them have been carefully studied by the Director and Assistant Director of the Study. From them we derived many of the concrete suggestions which are to be found in the body of the Study. The fact that so many individuals took a great deal of trouble to express their views convinced us of the timeliness of the Study, and creates in us the hope that the recommendations in the Study will be given serious consideration and, where deemed appropriate, acted upon.

Briefs from Individuals

Adams, R.A.
Department of Mathematics
University of British Columbia

Allan, G.F.
Actuary
Standard Life Assurance Co.

Altmann, A.
Department of Modern Languages
Simon Fraser University

Anthony, E.H.
Department of Zoology
University of Guelph

Antonelli, P.
Department of Mathematics
University of Alberta

Armstrong, K.W.
Bachelor of Education Program
University of Winnipeg

Baggs, I.
Department of Mathematics
University of Alberta

Banachewski, B.
Department of Mathematics
McMaster University

Barr, M.
Department of Mathematics
McGill University

Bartlett, W.
Operations Research Division
Air Canada

Beaumont, C.F.A.
Faculty of Mathematics
University of Waterloo

Beck, J.S.
Faculty of mathematics
University of Calgary

Blackwell, J.
Department of Applied Mathematics
University of Western Ontario

Bliss, L.C.
Department of Biology
University of Alberta

Boeckner, R.G.
Actuary Crown Life
Insurance Co.

Bromley, D.A.
Department of Physics
Yale University

Brown, J.D.
Faculty of Engineering
University of Western Ontario

Browne, P.
Alcan
Africa

Bulman-Fleming, S.
Faculty of Mathematics
University of Waterloo

Butz, E.
Department of Mathematics
University of Alberta

Campbell, L.L.
Department of Mathematics
Queen's University

Chaudry, M.D.
Department of Pol. & Eco. Science
Royal Military College

Chess, G.F.
Faculty of Engineering Science
University of Western Ontario

Colgan, P.
Department of Biology
Queen's University

Crapo, H.
Faculty of Mathematics
University of Waterloo

Cross, G.
Department of Mathematics
University of Calgary

Davis, C.
Department of Mathematics
University of Toronto

Davis, J.
Department of Mathematics
Queen's University

Demirdache, A.R.
Director
Ministry of State for Science
and Technology

Derome, J.R.
Departement de Physique
Universite de Montreal

Dobell, A.R.
Quantitative Analysis Course
Treasury Board Secretariat

Elrick, D.E.
Department of Land Resource Science
University of Guelph

Eames, W.
Department of Mathematics
Lakehead University

Fahidy, T.Z.
Department of Chemical Engineering
University of Waterloo

Ewen, B.
Past President
British Columbia Teachers Assoc.

Ferguson, G.A.
Department of Psychology
McGill University

Gold, A.G.
Department of Mathematics
University of Windsor

Gratwick, J.
Vice President
Canadian National Railways

Hellinell, J.
Department of Economics
University of British Columbia

Hermance, C.E.
Department of Mechanical Eng.
University of Waterloo

Hunka, S.
Faculty of Education
University of Alberta

Kaluzniacky, E.
Faculty of Administrative Studies
University of Manitooba

Kennedy, D.E.
Department of Mathematics
University of Victoria

Laird, P.G.
Department of Mathematics
University of Alberta

Lewis, T.
Department of Mathematics
University of Alberta

Friars, G.W.
Department of Animal & Poultry Science
University of Guelph

Graham, G.A.C.
Department of Mathematics
Simon Fraser University

Harrop, R.
Department of Mathematics
Simon Fraser University

Heuser, M.
Department of Mathematics
Vanier College

Hoechsman, K.
Department of Mathematics
University of British Columbia

Husain, T.
Department of Mathematics
McMaster University

Karvellas, P.H.
Deparment of Elect. Eng.
University of Alberta

Kretzschmann, M.H.
Superintendent of Comp. Systems
Falconbridge Nickel Mines Ltd.

Lamperti, J.
Department of Mathematics
Dartmouth College

McCartney, J.R.
Institute for Aerospace Studies
University of Toronto

McQueen, C.
Departement des Mathematiques
Universite Laval

McLaren, K.
Department of Mathematics
Pickering College

McTaggart-Cowan, P.D.
Executive Director
Science Council of Canada

Manohar, R.
Department of Mathematics
University of Saskatchewan

Matthieu, L.
Actuary
Alliance compagnie mutuelle d'assurance-vie

Mewhort, D.J.K.
Department of Psychology
Queen's University

Milner, E.C.
Department of Mathematics, Statistics & Computer Science
University of Calgary

Moore, J.C.G.
Department of Geology
MOunt Allison University

Northover, F.
Department of Mathematics
Carleton University

Ogilvie, J.C.
Director
Institute of Applied Statistics
University of Toronto

Pielou, E.C.
Department of Biology
Dalhousie University

Macki, J. & Muldowney, J.
Department of Mathematics
University of Alberta

Marzec, L.
Department of Mathematics
Mohawk College

Methot, J.C.
Department of Chemical Eng.
Laval University

Miller, D.R.
Department of Biomathematics
NRC Laboratories

Moore, E.
Faculty of Engineering and Applied Science
Memorial University

Morrison, F.E.
Associate Research Officer
Ottawa Board of Education

Nuttal, J.
Department of Physics
University of Western Ontario

Park, E.C.
Department of Math., Stat., and Comp. Science
University of Calgary

Probert R.L.
Faculty of Commerce
University of Saskatchewan

Ridge, H.L.
Faculty of Education
University of Toronto

Richardson I.W.
Faculty of Medicine
Dalhousie University

Robinson, G. de B.
Department of Mathematics
University of Toronto

Roach, M.R.
Department of Biophysics
University of Western Ontario

Rollerson L.G.
Actuary
Crown Life Insurance Co.

Robinson, P.
Department of Communication
Government of Canada

Routledge, R.
Department of Biology
Dalhousie University

Routledge, L.
High School Mathematics Teacher
Dartmouth, Nova Scotia

Rutherford, J.R.
Dupont (Canada) Ltd.
Kingston, Ontario

Royce, J.R.
Centre for Advanced Study in
Theoretical Pschology
University of Alberta

Schneider, W.G.
President
National Research Council

Scharff, A.R.
President
Montreal Actuaries Club

Selby, K.A.
Department of Civil Engineering
University of Toronto

Scroggie, G.A.
Educational Officer
Ontario Ministry of Education

Smith, J.T.
Department of Mathematics
Queen's University

Servranckx, R.
Department of Mathematics
University of Saskatchewan

Sprott, D.A.
Faculty of Mathematics
University of Waterloo

Smith, W.R.
Department of Mathematics
Dalhousie University

Sterling, T.D.
Director
Computing Science Program
Simon Fraser University

Stauffer, A.D.
Faculty of Science
York University

Taylor, J.
Department of Mathematics
McGill University

Tan, P.
Department of Mathematics
Carleton University

Tonks, R.S.
Director
College of Pharmacy
Dalhousie University

Taylor, P.D.
Department of Mathematics
Queen's University

Turner, R.
Department of Mathematics
University of Alberta

Van Brummelen, H.
Principal
Edmonton Christian High School

Voorhees, B.H.
Department of Mathematics
University of Alberta

Wheeler, D.
Education Solutions Inc.
New York

Wilson, P.
Canadian National Railways

Woods, R.G.
Department of Mathematics
and Astronomy
University of Manitoba

Worthington, A.G.
Department of Psychology
Trent University

Ursell, J.
Department of Mathematics
Queen's University

Vogt, E.
Department of Actuarial and Business
Science
University of Manitoba

Wehlau, W.H.
Department of Astronomy
University of Western Ontario

Williams, W.H.
Department of Mathematics
Simon Fraser University

Wilton, R.C.
Actuary
Dominion Life Assurance Co.

Woodside, W.
Department of Mathematics
Queen's University

Wright, G.P.
Department of Mathematics
University of Ottawa

Youngman, A.
Oakwood Collegiate Institute
Toronto, Ontario